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Stable Non-Supersymmetric Vacua in the Moduli Space of Non-Critical Superstrings

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We study a set of asymmetric deformations of non-critical superstring theories in various dimensions. The deformations arise as Kähler and complex structure deformations of an orthogonal two-torus comprising of a parallel and a transverse direction in the near-horizon geometry of NS5-branes. The resulting theories have the following intriguing features: Spacetime supersymmetry is broken in a continuous fashion and the masses of the lightest modes are lifted. In particular, no bulk or localized tachyons are generated in the non-supersymmetric vacua. We discuss the effects of these deformations in the context of the holographic duality between non-critical superstrings and Little String Theories and find solutions of rotating fivebranes in supergravity. We also comment on the generation of a one-loop cosmological constant and determine the effects of the one-loop backreaction to leading order.

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1. Introduction

String theory with broken supersymmetry exhibits a variety of interesting features. At tree-level, the theory can develop perturbative instabilities, which are signaled by the presence of tachyons, *i.e.* particles with negative mass squared, in the perturbative spectrum. The condensation of these modes is a time-dependent process driving the theory towards a more stable vacuum, where some amount of supersymmetry is usually restored. In recent years, much progress has been achieved in understanding such processes in open string theory (for reviews see [1,2]). A corresponding analysis of bulk tachyon dynamics is significantly more involved and a general treatment is still lacking (see, however, [3,4] and references therein). At higher loops, an effective potential is generated, which typically lifts some of the flat directions of the theory and produces a non-vanishing cosmological constant. Features like broken supersymmetry, time dependence and a non-vanishing cosmological constant are expected to be standard properties of any fundamental theory of the real world. Hence, it is imperative to understand these and other dynamical aspects of supersymmetry breaking in string theory.

In standard compactifications of string theory a successful resolution of the hierarchy problem requires that we break supersymmetry at a scale m_{SUSY} far below the string scale m_s . Usually, when we break supersymmetry at tree-level the supersymmetry breaking scale is closely tied to the compactification scale m_c . The Scherk-Schwarz mechanism is a typical example of this property. At weak g_s coupling the compactification scale is

itself comparable to the string scale which makes it difficult to generate a large separation of scales between m_{SUSY} and m_s [5,6]. Supersymmetry breaking at higher loops in perturbation theory or by non-perturbative effects, such as gaugino condensation, instantons *etc.* can avoid this problem and is very attractive for phenomenological applications, but takes us into the realm of strong coupling dynamics and will not be discussed here.

The simplest, and most obvious way, to obtain a small m_{SUSY}/m_s ratio and a correspondingly small one-loop cosmological constant, is to start with a supersymmetric vacuum and continuously turn on a modulus that breaks the supersymmetry. Thus, vacua in the neighborhood of the supersymmetric point would exhibit an arbitrarily small breaking of supersymmetry. There are, however, general arguments in standard string theory compactifications [7,8,6] that exclude this possibility. The Scherk-Schwarz supersymmetry breaking mechanism is again a nice example of the generic situation. Supersymmetry is explicitly broken at any finite compactification radius and is only restored when the radius becomes infinite, *i.e.* at infinite distance in the moduli space.

An interesting loophole to the general arguments of the previous paragraph can be found in string theories that live on asymptotically linear dilaton backgrounds. These are typically backgrounds of the general form

$$\mathbb{R}^{d-1,1} \times \mathbb{R}_\phi \times \mathcal{N} , \quad (1.1)$$

where $\mathbb{R}^{d-1,1}$ is the d -dimensional Minkowski spacetime, \mathbb{R}_ϕ is a linear dilaton direction labeled by ϕ and \mathcal{N} is a compact space. The string coupling g_s vanishes at the asymptotic boundary $\phi \rightarrow \infty$ and grows exponentially as we move towards smaller and smaller ϕ . With an even number of Minkowski spacetime dimensions d ,¹ a special class of solutions that preserves at least $2^{\frac{d}{2}+1}$ spacetime supersymmetries has been considered in [9]. In these solutions (1.1) takes the form

$$\mathbb{R}^{d-1,1} \times \mathbb{R}_\phi \times (S^1 \times \mathcal{M})/\Gamma , \quad (1.2)$$

where \mathcal{M} is the target space of a two-dimensional CFT with $\mathcal{N} = (2,2)$ worldsheet supersymmetry and Γ is a discrete group associated with the chiral GSO projection. String theory on (1.2) appears naturally in the near horizon limit of NS5-branes in type II string theory and defines the holographic dual of Little String Theory (LST) [10,11].

¹ $d = 2n$, with $n = 0, 1, \dots, 4$. $d = 8$ is the ten-dimensional critical string.

It is important to stress that the radius of the S^1 that appears in (1.2) is not arbitrary, but is fixed by the GSO projection in terms of the linear dilaton slope. Only at this special radius is string theory on (1.2) spacetime supersymmetric. Thus, a natural way to break the spacetime supersymmetry continuously (and evade the general arguments of [7,8,6]) is to turn on the modulus that changes the radius of the S^1 [9]. The resulting moduli space of vacua has been studied recently in [12], following earlier work on the $d = 0$ case in [13].

The stability properties of the vacua obtained by the above non-supersymmetric deformations was one of the main issues analyzed in [12]. In spacetimes of the form (1.1), there are in general two kinds of instabilities that can appear when we break the spacetime supersymmetry. One kind corresponds to delta function normalizable states propagating in the bulk of the linear dilaton throat \mathbb{R}_ϕ . Henceforth, we will refer to such instabilities as bulk tachyons. The other kind is characterized by normalizable states localized deep inside the strongly coupled region of the throat. We will refer to such instabilities as localized tachyons.

The analysis of [12] revealed that the stability properties of string theory in the moduli space of (1.2) depend crucially on the flat spacetime dimension d and the compact manifold \mathcal{M} . In the vicinity of the supersymmetric point and for $d = 2$ the theory is free of both bulk and localized tachyons. The same is true also for $d = 3$, an interesting odd d case that describes the near horizon region of a system of intersecting NS5-branes [14]. For $d = 4$ and $\mathcal{M} = 0$, a case that describes the decoupling limit of the conifold singularity, the theory exhibits both bulk and localized tachyons arbitrarily close to the supersymmetric point. Finally, for $d = 6$ and $\mathcal{M} = 0$ or $\mathcal{M} = \frac{SU(2)}{U(1)}$, the theory exhibits a localized tachyon, but no bulk tachyons in a finite region in the moduli space around the supersymmetric point.

In this paper, we show that there is a larger moduli space of non-supersymmetric vacua around the supersymmetric theory on (1.2). In order to explain the general situation, let us consider the following prototypical example. For $d = 6$, $\mathcal{M} = 0$ and $R_Y = Q = 1/\sqrt{2}$ (R_Y is the radius of the S^1 labeled here by Y and Q is the linear dilaton slope) the background (1.2) preserves sixteen supersymmetries and is relevant for the near-horizon dynamics of two parallel NS5-branes. Compactifying one of the worldvolume directions (let us call it X) preserves the same amount of supersymmetry and gives rise to the non-critical background

$$\mathbb{R}^{4,1} \times \mathbb{R}_\phi \times S_X^1 \times S_Y^1 . \quad (1.3)$$

From the five-dimensional point of view we can write the spacetime supercharges as

$$Q_\alpha^1 = \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2} + \frac{i}{2}(H_2 + H_3 - \sqrt{2}Y) + i\frac{\alpha}{2}(H_0 + H_1)}, Q_{\dot{\alpha}}^1 = \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2} - \frac{i}{2}(H_2 + H_3 - \sqrt{2}Y) + i\frac{\dot{\alpha}}{2}(H_0 - H_1)}, \quad (1.4)$$

$$Q_\alpha^2 = \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2} + \frac{i}{2}(-H_2 + H_3 - \sqrt{2}Y) + i\frac{\alpha}{2}(-H_0 + H_1)}, Q_{\dot{\alpha}}^2 = \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2} + \frac{i}{2}(H_2 - H_3 + \sqrt{2}Y) - i\frac{\dot{\alpha}}{2}(H_0 + H_1)}, \quad (1.5)$$

where $\alpha = \pm$, $\dot{\alpha} = \pm$ and H_2, H_3 are respectively the bosons bosonizing the worldsheet fermions (ψ^X, ψ^{x^4}) and (ψ^Y, ψ^ϕ) . More details on our conventions and the spacetime supersymmetry algebra can be found in the next section and appendix A. An additional equal number of spacetime supercharges $\bar{Q}_\alpha^i, \bar{Q}_{\dot{\alpha}}^i$ ($i = 1, 2$) arises from the right-moving sector.

The two-torus $S_X^1 \times S_Y^1$ of the asymptotic geometry (1.3) exhibits four independent exactly marginal deformations (the usual two complex and two Kähler structure deformations of the two-torus). The first is given by the worldsheet interaction $\int d^2z \partial X \bar{\partial} X$ and corresponds to the modulus that changes the compactification radius R_X . This modulus commutes with the spacetime supercharges (1.4), (1.5) and, as expected, does not break any spacetime supersymmetry. The second deformation is given by the worldsheet interaction $\int d^2z \partial Y \bar{\partial} Y$ and corresponds to the modulus that changes the S_Y^1 radius R_Y . This is precisely the deformation analyzed in [12]. As we see explicitly here, this modulus does not commute with any of the supercharges (1.4), (1.5) and breaks the spacetime supersymmetry completely. The remaining two-parameter family of deformations is given by the (asymmetric) worldsheet interaction

$$\delta\mathcal{S}_{(\lambda_+, \lambda_-)} \propto \int d^2z \left(\lambda_+ \mathcal{O}_+(z, \bar{z}) + \lambda_- \mathcal{O}_-(z, \bar{z}) \right), \quad (1.6)$$

where

$$\mathcal{O}_\pm = \partial X \bar{\partial} Y \pm \partial Y \bar{\partial} X. \quad (1.7)$$

This is a modulus that turns on a constant off-diagonal component of the metric and/or a constant B-field. As we explain in the main text, this perturbation can be viewed as giving an expectation value to the corresponding bulk gauge fields, *i.e.* turning on an appropriate Wilson line.

For general values of the deformation parameters λ_\pm with $|\lambda_+| \neq |\lambda_-|$ the worldsheet interaction $\delta\mathcal{S}_{(\lambda_+, \lambda_-)}$ does not commute with any of the spacetime supercharges (1.4), (1.5) and like $\int d^2z \partial Y \bar{\partial} Y$ it breaks the spacetime supersymmetry completely. On the

special one-dimensional submanifold $|\lambda_+| = |\lambda_-|$ the deformations break only half of the supersymmetry. For concreteness, we denote these special deformations as

$$\delta\mathcal{S}_\lambda \propto \frac{\lambda}{2} \int d^2z \left(\mathcal{O}_+ + \mathcal{O}_- \right) = \lambda \int d^2z \partial X \bar{\partial} Y, \quad (1.8)$$

$$\delta\mathcal{S}_{\bar{\lambda}} \propto \frac{\bar{\lambda}}{2} \int d^2z \left(\mathcal{O}_+ - \mathcal{O}_- \right) = \bar{\lambda} \int d^2z \partial Y \bar{\partial} X. \quad (1.9)$$

$\delta\mathcal{S}_\lambda$ preserves the eight supercharges $Q_\alpha^i, \bar{Q}_{\dot{\alpha}}^i$ arising from the left-moving sector of the worldsheet, and $\delta\mathcal{S}_{\bar{\lambda}}$ preserves the other eight supercharges $\bar{Q}_\alpha^i, Q_{\dot{\alpha}}^i$ arising from the right-moving sector.

It is worth mentioning that a set of asymmetric deformations, very similar to our $\delta\mathcal{S}_\lambda, \delta\mathcal{S}_{\bar{\lambda}}$ above, has been considered also in the past in the context of heterotic $SU(2)$ and $SL(2)$ WZW models [15,16]. There it was argued that an appropriate asymmetric deformation gives rise to a line of exact conformal field theories interpolating between the $SU(2)$ (or $SL(2)$) WZW model and the CFT that describes string propagation on the geometric coset S^2 (or AdS_2).

The main purpose of this paper is to analyze the effect of the general asymmetric deformation (1.6) on type II string theory on spacetimes of the form (1.2). For concreteness, we will focus on the $d = 6$ case with $\mathcal{M} = 0$ or $\mathcal{M} = SU(2)/U(1)$ and will compactify one of the flat spatial \mathbb{R}^{d-1} directions, which we will call in general X . We should emphasize, however, that similar results can be obtained for any other $d > 0$ and \mathcal{M} and for asymmetric deformations involving any of the flat $\mathbb{R}^{d-1,1}$ directions. Some of the possible extensions will be discussed briefly in section 6.

Special emphasis will be given to the stability properties of the moduli space that arises by turning on the asymmetric deformations. One of the striking results of our analysis is that, contrary to the symmetric $\int d^2z \partial Y \bar{\partial} Y$ deformation, the asymmetric deformations always give non-negative contributions to the masses of the lightest modes and bulk or localized tachyons do not appear. This surprising feature is independent of the dimension d and allows us to construct tree-level stable non-supersymmetric string theories with asymptotic linear dilaton directions. A detailed analysis of the effect of the asymmetric deformations on the spectrum of the theory appears in section 2.

Another interesting aspect of our analysis has to do with holography. As we mentioned earlier, string theory on (1.2) defines the holographic dual of a d -dimensional non-gravitational (and non-local) theory known as Little String Theory. Many aspects of

holography in this setting have been discussed in a series of papers [17-19]. It is known, in particular, how to map in the type IIB case a class of spacetime chiral primary states in the non-critical string description to low-energy gauge theory operators in the S-dual D5-brane gauge theory. It is interesting to identify the leading order effects of the asymmetric deformation (1.6) of the bulk theory in the dual D5-brane gauge theory. Using the general holographic prescription, the deformation amounts in the gauge theory to adding appropriate R-symmetry currents. This is explicitly verified in section 3 with a D5-brane probe analysis in the S-dual of the deformed non-critical string background. Furthermore, we will see that the results of the DBI analysis are in good agreement with the leading order spectrum deformations in section 2. A similar analysis for the symmetric deformation $\int d^2z \partial Y \bar{\partial} Y$ was performed in [12]. In addition, we find in general that the lightest modes in the bulk receive their first mass squared contribution at second order in the deformation parameter. On the D5-brane side, this implies a quadratic potential interaction, which is a non-chiral operator in a non-supersymmetric theory. The D5-brane probe analysis also reproduces this term.

In section 4 we explore the physics of the Higgs branch of the dual LST. At the supersymmetric point (*i.e* before the asymmetric deformation), we can regularize the strong coupling singularity of the backgrounds (1.2), by turning on the appropriate $\mathcal{N} = 2$ Liouville interaction on the worldsheet theory. In the language of NS5-branes this deformation takes us into the Higgs branch, where the NS5-branes are separated in an appropriate double scaling limit [18]. In the presence of the asymmetric deformation, the $\mathcal{N} = 2$ Liouville interaction is irrelevant (massive) on the worldsheet and the true exactly marginal Liouville interaction is time-dependent. In section 4, we find exact supergravity backgrounds in the large k limit, which describe the non-trivial rotation of k parallel NS5-branes in the presence of the asymmetric deformation. We deduce the precise form of these rotating solutions with a sequence of T-dualities and boosts. Similar solutions for $\int d^2z \partial Y \bar{\partial} Y$ appeared in [12].

In section 5, we discuss some of the features of the one-loop backreaction problem for the non-supersymmetric deformations and the generation of a one-loop cosmological constant. We explicitly find for the deformed CHS background the leading order correction coming from the one-loop backreaction. In section 6, we discuss some immediate extensions of our work and give an overview of the larger moduli space of string theory on (1.2). We conclude in section 7 with a brief summary and some interesting prospects. Two appendices are included summarizing our notation and some facts that are useful in the main text.

2. Deformations of the spectrum

In this section we define the asymmetric deformations of interest more generally and analyze their effect on the spectrum of the theory for a special class of backgrounds of the form (1.2) with $d = 6$ and $\mathcal{M} = SU(2)/U(1)$. Similar results can be obtained also in other dimensions and with different compact spaces \mathcal{M} . Possible extensions will be discussed in section 6.

2.1. Setting the stage

A specific example of the general form (1.1) is the CHS background [20]

$$\mathbb{R}^{5,1} \times \mathbb{R}_\phi \times SU(2)_k, \quad (2.1)$$

which appears in the near horizon geometry of k parallel NS5-branes. The linear dilaton slope depends on the number of NS5-branes via the relation²

$$Q = 1/\sqrt{k}. \quad (2.2)$$

For what follows, it will be convenient to single out one of the flat directions labeled by X and compactify it with an arbitrary radius R_X . We will denote the remaining flat directions as x^μ with $\mu = 0, 1, \dots, 4$. In addition, the worldsheet theory on (2.1) comprises of a set of free real fermions ψ^μ , ψ^X , ψ^ϕ and the $\mathcal{N} = 1$ supersymmetric WZW model $SU(2)_k$. The latter can be written as a direct sum of the bosonic $SU(2)_{k-2}$ WZW model and three free fermions χ^3, χ^\pm .

The CHS background (2.1) can also be recast as

$$\mathbb{R}^{5,1} \times \mathbb{R}_\phi \times \left(S_k^1 \times \frac{SU(2)_k}{U(1)} \right) / \mathbb{Z}_k \quad (2.3)$$

by using the well-known decomposition of the $SU(2)_k$

$$SU(2)_k = \left(S_k^1 \times \frac{SU(2)_k}{U(1)} \right) / \mathbb{Z}_k \quad (2.4)$$

in terms of the $\mathcal{N} = 2$ minimal model $SU(2)_k/U(1)$. Hence, (2.3) is a special case of the general solution (1.2) with $\mathcal{M}_k = SU(2)_k/U(1)$ and $\Gamma = \mathbb{Z}_k$. Notice that for $k = 2$ the minimal model \mathcal{M}_2 is empty and (2.3) becomes simply

$$\mathbb{R}^{5,1} \times \mathbb{R}_\phi \times S^1. \quad (2.5)$$

² We use the convention $\alpha' = 1$ in this paper. Our notation is summarized in appendix A.

At the supersymmetric point, the radius of the S^1 in (2.3) is fixed by the GSO projection to be Q or $1/Q$ (the two are related by T-duality). By changing this radius we break the spacetime supersymmetry and move away from the supersymmetric point in the moduli space. The properties of this deformation were analyzed extensively in [12]. The worldsheet modulus that is responsible for this effect can be written as

$$\delta\mathcal{S} = \lambda \int d^2z \, \partial Y \bar{\partial} Y , \quad (2.6)$$

where Y is the boson that labels the S^1 in (2.3). In the $SU(2)$ formulation (2.1) we can think of Y as the boson that bosonizes the total $K_3^{(tot)}$ current of the $SU(2)_k$ WZW model, *i.e.*

$$K_3^{(tot)} = K_3 + \chi^+ \chi^- = i\sqrt{k}\partial Y . \quad (2.7)$$

In this expression K_3 is the Cartan current of the *bosonic* $SU(2)_{k-2}$ current algebra. Thus, in the CHS geometry (2.1) we can write the deformation (2.6) as

$$\delta\mathcal{S} = -\lambda \int d^2z \, G_{-1/2} \bar{G}_{-1/2} \chi^3 \bar{\chi}^3 = -Q^2 \lambda \int d^2z \, K_3^{(tot)}(z) \bar{K}_3^{(tot)}(\bar{z}) = \lambda \int d^2z \, \partial Y \bar{\partial} Y \quad (2.8)$$

where $G_{-1/2}$, $\bar{G}_{-1/2}$ are the fermionic generators of the $\mathcal{N} = 1$ worldsheet supersymmetry algebra (see appendix A). This particular current-current deformation has been discussed in various papers in the past [21–25]. From the spacetime point of view, it is a deformation that squashes the three-sphere transverse to the fivebranes and since it does not commute with the spacetime supercharges it also breaks the spacetime supersymmetry explicitly.

In this paper, we are interested in another set of current-current deformations, which are left-right asymmetric and couple the parallel and transverse directions of the fivebranes in an interesting fashion. They have the general form³

$$\begin{aligned} \delta\mathcal{S}_{(\tilde{\lambda}_+, \tilde{\lambda}_-)} &= -\frac{\tilde{\lambda}_+ + \tilde{\lambda}_-}{4\pi} \int d^2z \, G_{-1/2} \bar{G}_{-1/2} \psi^X \bar{\chi}^3 - \frac{\tilde{\lambda}_+ - \tilde{\lambda}_-}{4\pi} \int d^2z \, G_{-1/2} \bar{G}_{-1/2} \chi^3 \bar{\psi}^X = \\ &= \frac{1}{4\pi} \int d^2z \left(\tilde{\lambda}_+ \mathcal{O}_+(z, \bar{z}) + \tilde{\lambda}_- \mathcal{O}_-(z, \bar{z}) \right) , \end{aligned} \quad (2.9)$$

with

$$\mathcal{O}_{\pm} = \partial X \bar{\partial} Y \pm \partial Y \bar{\partial} X . \quad (2.10)$$

³ Anticipating the discussion of the next section it is convenient to label the deformation parameters here as $\tilde{\lambda}_{\pm}$.

The $\frac{1}{4\pi}$ factor has been inserted for later convenience and the bosons X, Y satisfy the periodicity conditions $X \sim X + 2\pi R_X$, $Y \sim Y + 2\pi R_Y$. As we mentioned in the introduction, for generic values of the deformation parameters $\tilde{\lambda}_\pm$ the modulus (2.9) breaks the spacetime supersymmetry completely. The deformation preserves half of the spacetime supersymmetry when $|\tilde{\lambda}_+| = |\tilde{\lambda}_-|$ (see appendix A for the explicit form of the spacetime supercharges). Our aim in this section is to determine the effect of the deformations (2.9), (2.10) on the spectrum of the supersymmetric theory.

The vertex operators that appear in (2.9) express the x -components $(\mathcal{A}_L)_x^3$ and $(\mathcal{A}_R)_x^3$ of the $SU(2)_{L,R}$ bulk gauge fields $(\mathcal{A}_{L,R})_\mu^a$ ($\mu = 0 \dots 5$, $a = 3, \pm$). As a consequence, the asymmetric deformation (2.9) can also be written as

$$\delta\mathcal{S}_{(\tilde{\lambda}_+, \tilde{\lambda}_-)} \propto \int d^2z \left(\tilde{\lambda}_- [(\mathcal{A}_R)_x^3 - (\mathcal{A}_L)_x^3] + \tilde{\lambda}_+ [(\mathcal{A}_R)_x^3 + (\mathcal{A}_L)_x^3] \right) \quad (2.11)$$

and hence can be viewed as giving an expectation value to these gauge field components.

2.2. $O(2,2)$ deformations of the spectrum

Let us begin by recalling the main features of the spectrum at the supersymmetric point. The $k = 2$ case (2.5) is simpler and we review it first. In the linear dilaton geometry (2.5) the general vertex operator (without any string oscillators) takes the form

$$\mathcal{V} = e^{-(1-\frac{\alpha}{2})\varphi - (1-\frac{\bar{\alpha}}{2})\bar{\varphi}} e^{i \sum_{i=0}^3 (s_i H_i + \bar{s}_i \bar{H}_i)} e^{i(pX + \bar{p}\bar{X}) + i(qY + \bar{q}\bar{Y})} e^{ip_\mu x^\mu} e^{\beta\phi}, \quad (2.12)$$

where $\alpha = 0$ stands for the NS sector and $\alpha = 1$ for the R sector. φ is the standard (β, γ) superghost of the fermionic string and H_i ($i = 0, 1, 2, 3$) are four bosons bosonizing the eight free worldsheet fermions of the theory (more details can be found in appendix A).

Following the notation of [6] we denote the general closed string sector of the theory by

$$(\alpha, F, \bar{\alpha}, \bar{F}), \quad (2.13)$$

where $\alpha, \bar{\alpha}$ are the labels appearing in (2.12) and F, \bar{F} are the left- and right-moving worldsheet fermion numbers modulo two. In the $d = 6$ case that we study here $F = 0, 1$ in the NS sector and $\pm \frac{1}{2}$ in the Ramond sector.

The allowed quantum numbers in the supersymmetric theory are determined by the chiral GSO projection (*i.e.* the requirement of locality of the generic vertex operator (2.12))

with the spacetime supercharges), the mutual locality conditions and the physical state conditions. In the present case, the mutual locality condition reads

$$F_1\alpha_2 - F_2\alpha_1 - \bar{F}_1\bar{\alpha}_2 + \bar{F}_2\bar{\alpha}_1 - \frac{1}{2}(\alpha_1\alpha_2 - \bar{\alpha}_1\bar{\alpha}_2) + 2(n_1w_2 + n_2w_1) \in 2\mathbb{Z} , \quad (2.14)$$

where n, w are the momentum and winding quantum numbers in the Y direction, *i.e.* in (2.12)

$$q = \frac{n}{R_Y} + wR_Y , \quad \bar{q} = \frac{n}{R_Y} - wR_Y . \quad (2.15)$$

At the supersymmetric point $R_Y = Q = 1/\sqrt{2}$. The allowed sectors and states can be found in [26,27,12].

For $k > 2$ the analog of the general vertex operator (2.12) is

$$\mathcal{V} = e^{-(1-\frac{\alpha}{2})\varphi - (1-\frac{\bar{\alpha}}{2})\bar{\varphi}} e^{i\sum_{i=0}^3 (s_i H_i + \bar{s}_i \bar{H}_i)} e^{i(pX + \bar{p}\bar{X}) + i(qY + \bar{q}\bar{Y})} e^{ip_\mu x^\mu} e^{\beta\phi} \Phi_{j+1, m, \bar{m}}, \quad (2.16)$$

where $\Phi_{j+1, m, \bar{m}}$ is a primary vertex operator of the $\mathcal{N} = 2$ supersymmetric minimal model $\mathcal{M}_k = SU(2)_k/U(1)$. The minimal model quantum numbers (j, m, \bar{m}) can take the values $j = 0, \frac{1}{2}, \dots, \frac{k}{2} - 1$, $m, \bar{m} = -j - 1, -j, -j + 1, \dots, j + 1$. The latter are intimately tied to the momentum and winding quantum numbers n, w along Y through the chiral GSO projection. n and w are still given by (2.15), but now $R_Y = Q = 1/\sqrt{k}$ and in terms of m, \bar{m}

$$q = 2Qm , \quad \bar{q} = 2Q\bar{m} . \quad (2.17)$$

Notice that the GSO projection allows for a fractional momentum quantum number $n \in \mathbb{Z}/k$.

The (five-dimensional) mass of the corresponding spacetime modes can be determined in the following manner. If we denote by h the (say, left-moving) scaling dimension of the generic vertex operator \mathcal{V} in (2.16) at five-dimensional momentum $p_\mu = 0$, we can write the mass-shell condition as

$$h - \frac{1}{4}M^2 = 1 \Leftrightarrow M^2 = 4(h - 1) , \quad (2.18)$$

where $M^2 = -p_\mu p^\mu$ is the five-dimensional mass squared. Notice that we do not have to deal separately with the left- and right-moving scaling dimensions, because one of the physical state conditions is $L_0 = \bar{L}_0$. Hence, in order to determine the effect of the asymmetric deformations (2.9) on the spectrum of the theory, we need to determine how it affects the scaling dimensions h . For example, if we have initially a massless mode and the

asymmetric deformation gives $\delta h < 0$, then a tachyon will appear in the deformed theory. This tachyon will be a localized tachyon if the quantum number β in (2.16) is real, or a bulk tachyon if $\beta = -\frac{Q}{2} + is$, $s \in \mathbb{R}$.

Determining the effect of the asymmetric deformations on the scaling dimensions h is a straightforward exercise, because the general deformation (2.9) acts only on the free (X, Y) part the worldsheet theory. Then, essentially we have to find the scaling dimensions of the vertex operators

$$e^{i(pX + \bar{p}\bar{X}) + i(qY + \bar{q}\bar{Y})} \quad (2.19)$$

under a general $O(2, 2)$ deformation of the orthogonal torus $S_X^1 \times S_Y^1$. A general expression for these scaling dimensions is known.

Consider the general two-torus with metric and B -field

$$ds^2 = G_{xx}dx^2 + G_{yy}dy^2 + 2G_{xy}dxdy, \quad B = B_{xy}dx \wedge dy. \quad (2.20)$$

where $X = R_X x$, $Y = R_Y y$. It is convenient to arrange the four real data $G_{xx}, G_{yy}, G_{xy}, B_{xy}$ in two complex parameters ρ and τ in the following manner

$$\tau \equiv \tau_1 + i\tau_2 = \frac{G_{xy}}{G_{yy}} + i\frac{\sqrt{G}}{G_{yy}}, \quad (2.21)$$

$$\rho \equiv \rho_1 + i\rho_2 = B_{xy} + i\sqrt{G}, \quad (2.22)$$

where $G = G_{xx}G_{yy} - G_{xy}^2$. The (left, right) scaling dimensions (h, \bar{h}) of a vertex operator with momenta n_x, n_y and windings w_x, w_y are in this background given by the following compact formulae (see *e.g.* [28])

$$h = \frac{1}{4\rho_2\tau_2} \left| (n_x - \tau n_y) - \rho(w_y + \tau w_x) \right|^2, \quad (2.23)$$

$$\bar{h} = \frac{1}{4\rho_2\tau_2} \left| (n_x - \tau n_y) - \bar{\rho}(w_y + \tau w_x) \right|^2. \quad (2.24)$$

For the deformation (2.9) of the diagonal torus we have

$$G_{xx} = R_X^2, \quad G_{yy} = R_Y^2, \quad G_{xy} = \tilde{\lambda}_+ R_X R_Y, \quad B_{xy} = \tilde{\lambda}_- R_X R_Y, \quad (2.25)$$

or

$$\tau = \frac{R_X}{R_Y} \left(\tilde{\lambda}_+ + i\sqrt{1 - \tilde{\lambda}_+^2} \right), \quad \rho = R_X R_Y \left(\tilde{\lambda}_- + i\sqrt{1 - \tilde{\lambda}_+^2} \right). \quad (2.26)$$

R_X is arbitrary and $R_Y = Q$. The latter has been fixed by the GSO projection at the supersymmetric point. By setting

$$\begin{aligned} p_n &= \frac{n_x}{R_X}, & p_w &= w_x R_X \\ q_n &= \frac{n_y}{R_Y}, & q_w &= w_y R_Y \end{aligned} \quad (2.27)$$

we find for the scaling dimensions (h, \bar{h}) the following expressions

$$\begin{aligned} h &= \frac{1}{8(1 + \tilde{\lambda}_+)} \left[p_n + q_n + (1 + \tilde{\lambda}_+ + \tilde{\lambda}_-) p_w + (1 + \tilde{\lambda}_+ - \tilde{\lambda}_-) q_w \right]^2 + \\ &+ \frac{1}{8(1 - \tilde{\lambda}_+)} \left[p_n - q_n + (1 - \tilde{\lambda}_+ - \tilde{\lambda}_-) p_w - (1 - \tilde{\lambda}_+ + \tilde{\lambda}_-) q_w \right]^2, \end{aligned} \quad (2.28)$$

$$\begin{aligned} \bar{h} &= \frac{1}{8(1 + \tilde{\lambda}_+)} \left[p_n + q_n - (1 + \tilde{\lambda}_+ - \tilde{\lambda}_-) p_w - (1 + \tilde{\lambda}_+ + \tilde{\lambda}_-) q_w \right]^2 + \\ &+ \frac{1}{8(1 - \tilde{\lambda}_+)} \left[p_n - q_n - (1 - \tilde{\lambda}_+ + \tilde{\lambda}_-) p_w + (1 - \tilde{\lambda}_+ - \tilde{\lambda}_-) q_w \right]^2. \end{aligned} \quad (2.29)$$

It follows that the difference $h - \bar{h} = p_n p_w + q_n q_w$ is $\tilde{\lambda}_\pm$ -independent, so the physical state condition $L_0 = \bar{L}_0$ will continue to hold after the deformation, if we impose it from the beginning.

In general, the scaling dimensions (2.28), (2.29) will become infinite at the boundary values $\tilde{\lambda}_+ = \pm 1$ and the deformation terminates there. The existence of a boundary value is an artifact of the particular parametrization of the moduli space that we are using. We will encounter these boundary values again in the next section, where we derive the effect of the asymmetric deformations on the CHS geometry (2.1). The moduli of asymmetric deformations is summarized in Figure 1.

The lightest modes of the theory have vanishing momentum and winding in the flat direction X . Some of these modes are actually massless at the supersymmetric point and as an important check of stability we need to verify how these masses are shifted by the supersymmetry breaking deformation. Setting $p_n = p_w = 0$ in (2.28), (2.29) we find

$$h = h_0 + \frac{1}{4(1 - \tilde{\lambda}_+^2)} \left(\tilde{\lambda}_+ q_n + \tilde{\lambda}_- q_w \right)^2, \quad (2.30)$$

where h_0 is the undeformed scaling dimension. Hence, we see that the mass squared of these modes receives always a non-negative contribution and that no bulk or localized tachyons are generated after the deformation. It is also interesting to note that the leading

order deformation of the scaling dimensions appears in general at second order in the deformation parameters and that a positive mass shift occurs also for the supersymmetric deformations $|\tilde{\lambda}_+| = |\tilde{\lambda}_-|$.

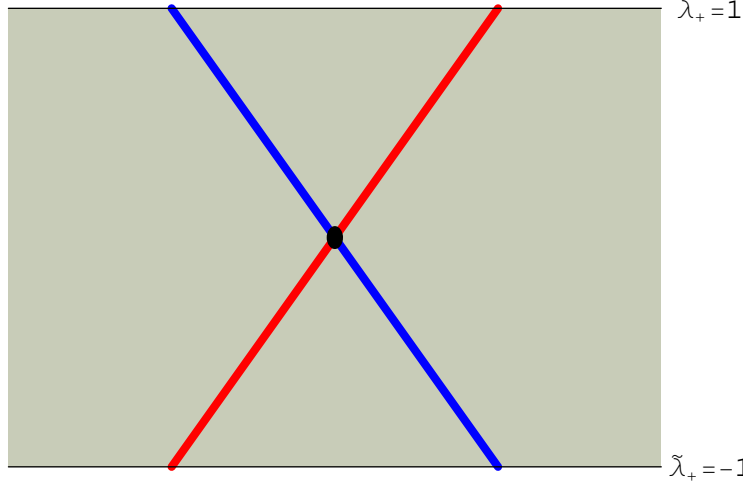


Figure 1. The $(\tilde{\lambda}_-, \tilde{\lambda}_+)$ plane of asymmetric deformations. The shaded region represents the allowed deformations. The generic point in this moduli space is a non-supersymmetric theory. The blue and red lines represent the deformations with $|\tilde{\lambda}_+| = |\tilde{\lambda}_-|$ that preserve half of the spacetime supersymmetry. The point at the center represents the undeformed type II theory with sixteen supercharges.

It is instructive to compute the small $\tilde{\lambda}_\pm$ expansion of the scaling dimensions (2.28), (2.29) with generic values for the X momenta. Expanding the general result up to second order in the deformation parameters we find

$$h = h_0 + \frac{\tilde{\lambda}_+}{2}(p_w q_w - p_n q_n) + \frac{\tilde{\lambda}_-}{2}(p_w q_n - q_w p_n) + \frac{1}{4} \left[\tilde{\lambda}_+^2 (p_n^2 + q_n^2) + \tilde{\lambda}_-^2 (p_w^2 + q_w^2) - 2\tilde{\lambda}_+ \tilde{\lambda}_- (p_w p_n - q_w q_n) \right]. \quad (2.31)$$

As a further check, we have verified this formula with an explicit computation of two-point functions in the perturbed theory. In the next section, we will discuss the effects of the deformation in the low-energy limit of the dual LST by performing a D5-brane probe analysis in the S-dual of the deformed CHS solution. We will find good agreement with (2.31), thus achieving a non-trivial test of holography in a non-supersymmetric context.

As a final comment on the stability of the deformed theories notice that the masses of the modes with non-vanishing p_n, p_w momenta can be shifted up or down. This is already

clear at the leading linear order in (2.31). Hence one might worry that under a finite deformation such a mode, although initially massive, can come down enough to become a tachyon. For example, this may happen if the difference

$$\delta h = h(\tilde{\lambda}_+, \tilde{\lambda}_-) \big|_{p_n, p_w} - h(0, 0) \big|_{p_n = p_w = 0} \quad (2.32)$$

is sufficiently negative. We have verified that $\delta h \geq 0$ for modes with vanishing winding $p_w = 0$. In the more general case, we have not been able to verify conclusively if a tachyon appears after a finite deformation or not. Certainly, such a tachyon cannot appear for the special deformations $|\tilde{\lambda}_+| = |\tilde{\lambda}_-|$ that preserve half of the original supersymmetry.

3. Asymmetric deformations of the holographic dual

As we mentioned earlier, the asymptotically linear dilaton theories (1.2) appear naturally in the near-horizon region of NS5-brane configurations and provide the holographic dual of the non-gravitational Little String Theory that lives on the fivebranes. For the special case (2.1), the appropriate configuration consists of k coincident parallel NS5-branes that live deep inside the strongly coupled region of the throat. In the type IIB case, the low-energy dynamics of this strongly coupled system is captured by the six-dimensional $\mathcal{N} = 1$ $SU(k)$ super-Yang Mills (SYM) theory that lives on the S-dual configuration of k parallel D5-branes. As in the usual AdS/CFT correspondence, many aspects of this holographic relation between bulk and boundary data are known [17-19]. There is, in particular, a well-established dictionary between spacetime chiral primary operators in the dual low energy SYM theory and corresponding observables in the non-critical superstring on (2.1). In this section, we analyze the effect of the non-supersymmetric deformation on the dual low energy gauge theory with a D5-brane probe analysis in the S-dual of the deformed CHS solution. We work in the large k limit where the spacetime curvatures are small. For completeness, we give here the relevant part of the type IIB supergravity action (in the string frame)

$$\mathcal{S}_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[e^{-2\Phi} \left(R + 4(\partial\Phi)^2 - \frac{1}{12} F_3^2 \right) - \frac{1}{12} G_3^2 \right] \quad (3.1)$$

where F_3 and G_3 are the NSNS and RR three-form field strengths respectively.

3.1. A D5-brane probe analysis

The CHS background of k parallel NS5-branes has the form⁴

$$\begin{aligned} ds^2 &= dx_{||}^2 + dx^2 + d\phi^2 + k(d\theta^2 + \sin^2 \theta d\phi_1^2 + \cos^2 \theta d\phi_2^2) , \\ B &= k \cos^2 \theta d\phi_1 \wedge d\phi_2 , \quad g_s^2 = e^{-2Q\phi} , \end{aligned} \quad (3.2)$$

where $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq \phi_1, \phi_2 \leq 2\pi$ and $Q = 1/\sqrt{k}$. $dx_{||}^2$ is the metric element for the five worldvolume directions x^μ , $\mu = 0, 1, \dots, 4$. We have singled out the sixth worldvolume direction $x \sim x + 2\pi R_X$, which will take part in the deformation.

With a T-duality transformation along the ϕ_2 direction we obtain the equivalent background⁵

$$ds^2 = dx_{||}^2 + dx^2 + d\phi^2 + k(d\theta^2 + d\phi_1^2) + 2d\phi_1 d\phi_2 + \frac{1}{k \cos^2 \theta} d\phi_2^2 , \quad B = 0 \quad (3.3)$$

and the appropriate dilaton. By defining the new coordinates [29]

$$\tilde{\phi}_1 = \phi_1 + \frac{\phi_2}{k} , \quad \tilde{\phi}_2 = \frac{\phi_2}{k} \quad (3.4)$$

we can recast (3.3) as

$$ds^2 = dx_{||}^2 + dx^2 + d\phi^2 + k(d\theta^2 + d\tilde{\phi}_1^2 + \tan^2 \theta d\tilde{\phi}_2^2) , \quad B = 0 . \quad (3.5)$$

Notice that the new variables $(\tilde{\phi}_1, \tilde{\phi}_2)$ are identified under

$$(\tilde{\phi}_1, \tilde{\phi}_2) \sim \left(\tilde{\phi}_1 + \frac{2\pi}{k}, \tilde{\phi}_2 + \frac{2\pi}{k} \right) . \quad (3.6)$$

The resulting background is none other than

$$\mathbb{R}^{4,1} \times S_x^1 \times \left(S_{\tilde{\phi}_1}^1 \times \frac{SU(2)_k}{U(1)} \right) / \mathbb{Z}_k , \quad (3.7)$$

which is the natural frame for the deformations (2.9) written in terms of the bosons X, Y . In terms of the variables $x, \tilde{\phi}_1$ used above we have the identifications⁶

$$x = X , \quad \tilde{\phi}_1 = \frac{Y}{\sqrt{k}} \quad (3.8)$$

⁴ With α' reinstated $g_s^2 = e^{-2Q\phi/\alpha'}$.

⁵ For convenience, we have summarized the general Buscher rules of T-duality in appendix B.

⁶ Note that the x used here is not the same as the x appearing in (2.20) .

The $\lambda_+ \mathcal{O}_+ + \lambda_- \mathcal{O}_-$ deformation takes the following form in the frame of (3.5)⁷

$$\begin{aligned} ds^2 &= dx^2 + k(d\theta^2 + d\tilde{\phi}_1^2 + \tan^2 \theta d\tilde{\phi}_2^2) + 2\lambda_+ dx d\tilde{\phi}_1^2 \\ B &= \lambda_- dx \wedge d\tilde{\phi}_1 . \end{aligned} \quad (3.9)$$

Using the identification (3.8) and comparing with (2.25) we deduce the relation

$$\lambda_{\pm} = \sqrt{k} \tilde{\lambda}_{\pm} \quad (3.10)$$

between the deformation parameter λ_{\pm} used above and $\tilde{\lambda}_{\pm}$ used in section 2.

Going back to the (ϕ_1, ϕ_2) coordinate system we find

$$\begin{aligned} ds^2 &= dx^2 + k(d\theta^2 + d\phi_1^2) + 2d\phi_1 d\phi_2 + \frac{1}{k \cos^2 \theta} d\phi_2^2 + 2\lambda_+ dx \left(d\phi_1 + \frac{d\phi_2}{k} \right) , \\ B &= \lambda_- dx \wedge \left(d\phi_1 + \frac{d\phi_2}{k} \right) . \end{aligned} \quad (3.11)$$

T-dualizing back along ϕ_2 , using the general rules of appendix B, we get the final form of the deformed near-horizon NS5-brane background

$$\begin{aligned} ds^2 &= dx_{||}^2 + \left(1 + \frac{\lambda_-^2 - \lambda_+^2}{k} \cos^2 \theta \right) dx^2 + d\phi^2 + k(d\theta^2 + \sin^2 \theta d\phi_1^2 + \cos^2 \theta d\phi_2^2) + \\ &+ 2\lambda_+ \sin^2 \theta dx d\phi_1 + 2\lambda_- \cos^2 \theta dx d\phi_2 , \end{aligned} \quad (3.12)$$

$$B = k \cos^2 \theta d\phi_1 \wedge d\phi_2 + \lambda_- \sin^2 \theta dx \wedge d\phi_1 + \lambda_+ \cos^2 \theta dx \wedge d\phi_2 , \quad (3.13)$$

$$g_s^2 = e^{-2Q\phi} \left(1 - \frac{\lambda_+^2}{k} \right) . \quad (3.14)$$

This solution, which is exact in the deformation parameters λ_{\pm} , exhibits a quadratic contribution to the dx^2 part of the metric and the dilaton. The quadratic contribution to the metric vanishes when $\lambda_+^2 = \lambda_-^2$, which is the case of the half supersymmetry preserving deformations. For the non-supersymmetric deformations the determinant of the metric vanishes identically at the boundary values $\lambda_+ = \pm \sqrt{k}$. Using the relation (3.10), we see that these are precisely the boundary values for $\tilde{\lambda}_+$ that we found in the previous section, when we computed the scaling dimensions.

⁷ In what follows, we momentarily omit the trivial $dx_{||}^2 + d\phi^2$ part of the metric, which does not participate in the deformation.

The S-dual D5-brane background corresponding to (3.12) - (3.14) reads

$$ds^2 = g_s \left[dx_{||}^2 + \left(1 + \frac{\lambda_-^2 - \lambda_+^2}{k} \cos^2 \theta \right) dx^2 + d\phi^2 + k(d\theta^2 + \sin^2 \theta d\phi_1^2 + \cos^2 \theta d\phi_2^2) + \right. \\ \left. + 2\lambda_+ \sin^2 \theta dx d\phi_1 + 2\lambda_- \cos^2 \theta dx d\phi_2 \right], \quad (3.15)$$

$$G_3 = 2 \cos \theta \sin \theta \left(-k d\phi_1 \wedge d\phi_2 \wedge d\theta + \lambda_- dx \wedge d\phi_1 \wedge d\theta - \lambda_+ dx \wedge d\phi_2 \wedge d\theta \right), \quad (3.16)$$

with

$$g_s^2 = e^{2Q\phi} \left(1 - \frac{\lambda_+^2}{k} \right)^{-1} \quad (3.17)$$

and $G_3 = dC_2$ the RR three-form field strength of the S-dual C_2 -field.

The D5-branes couple electrically to the Hodge-dual of G_3 . This is a seven-form $G_7 = *G_3$ with components

$$G_{\mu_1 \dots \mu_7} = \sqrt{g} g^{\nu_1 \rho_1} g^{\nu_2 \rho_2} g^{\nu_3 \rho_3} G_{\rho_1 \rho_2 \rho_3} \epsilon_{\nu_1 \nu_2 \nu_3 \mu_1 \dots \mu_7} = \\ = \sqrt{g} \left(g^{\nu_1 \phi_1} g^{\nu_2 \phi_2} g^{\nu_3 \theta} G_{\phi_1 \phi_2 \theta} + g^{\nu_1 x} g^{\nu_2 \phi_1} g^{\nu_3 \theta} G_{x \phi_1 \theta} + g^{\nu_1 x} g^{\nu_2 \phi_2} g^{\nu_3 \theta} G_{x \phi_2 \theta} \right) \epsilon_{\nu_1 \nu_2 \nu_3 \mu_1 \dots \mu_7}. \quad (3.18)$$

In our case the non-vanishing components of this tensor are $G_{01234x\phi}$, $G_{01234\phi_1\phi}$, $G_{01234\phi_2\phi}$. Only the first will play a rôle in the DBI analysis and by straightforward computation we find

$$G_{01234x\phi} = -2Q e^{2Q\phi} \left(1 - \frac{\lambda_+^2}{k} \right)^{-\frac{1}{2}}. \quad (3.19)$$

The corresponding C_6 potential component is

$$C_{01234x} = -e^{2Q\phi} \left(1 - \frac{\lambda_+^2}{k} \right)^{-\frac{1}{2}}. \quad (3.20)$$

All the pieces are now in place to proceed with the DBI analysis of a D5-brane that moves in the background (3.15) - (3.17). The DBI+WZ action (up to overall normalization) of the D5-branes is

$$\mathcal{S}_{D5} = \int d^6 \xi \frac{1}{g_s} \sqrt{-\det(G_{AB} + F_{AB})} + \int d^6 \xi C_6, \quad (3.21)$$

where ξ^A ($A, B = 0, 1, \dots, 5$) are the worldvolume directions,

$$G_{AB} = g_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^A} \frac{\partial X^\nu}{\partial \xi^B} \quad (3.22)$$

is the induced metric and F_{AB} the gauge field on the D5-brane. We will use the static gauge

$$X^a = \xi^a, \quad a = 0, 1, \dots, 4, \quad x = \xi^5 \quad (3.23)$$

and assume that the transverse coordinates ϕ, θ are ξ -independent. The coordinates ϕ_1 and ϕ_2 will depend in principle on ξ^A . Hence, the induced metric components are

$$G_{ab} = g_s \left(\eta_{ab} + k \sin^2 \theta \partial_a \phi_1 \partial_b \phi_1 + k \cos^2 \theta \partial_a \phi_2 \partial_b \phi_2 \right), \quad (3.24)$$

$$G_{xx} = g_s \left(1 + \frac{\lambda_-^2 - \lambda_+^2}{k} \cos^2 \theta + k \sin^2 \theta (\partial_x \phi_1)^2 + k \cos^2 \theta (\partial_x \phi_2)^2 + 2\lambda_+ \sin^2 \theta \partial_x \phi_1 + 2\lambda_- \cos^2 \theta \partial_x \phi_2 \right), \quad (3.25)$$

$$G_{xa} = g_s \left(k \sin^2 \theta \partial_x \phi_1 \partial_a \phi_1 + k \cos^2 \theta \partial_x \phi_2 \partial_a \phi_2 + \lambda_+ \sin^2 \theta \partial_a \phi_1 + \lambda_- \cos^2 \theta \partial_a \phi_2 \right), \quad (3.26)$$

with $a, b = 0, 1, \dots, 4$. The WZ term in (3.21) is

$$\int d^6 \xi \, C_6 = \int d^5 x_{||} dx \, C_{01234x}. \quad (3.27)$$

For simplicity, let us set $F_{AB} = 0$ and drop the kinetic and friction terms, *i.e.* those terms that involve x^a -derivatives ($a = 0, 1, \dots, 4$). Then, up to leading order in λ_{\pm} , we find the D5-brane Lagrangian

$$\mathcal{L}_{\text{D5}} = \mathcal{L}_{\text{D5}} \Big|_{\lambda_{\pm}=0} + \frac{\lambda_+ \sin^2 \theta \partial_x \phi_1 + \lambda_- \cos^2 \theta \partial_x \phi_2}{\sqrt{1 + k \sin^2 \theta (\partial_x \phi_1)^2 + k \cos^2 \theta (\partial_x \phi_2)^2}} e^{2Q\phi} + \mathcal{O}(\lambda_{\pm}^2). \quad (3.28)$$

The six-dimensional super Yang-Mills theory that lives on the D5-branes has four scalar fields in the adjoint of the gauge group. Before the deformation, the theory has a moduli space parametrized by the vacuum expectation values of these scalar fields, which simply encode the positions of the fivebranes in the four transverse directions. Let us combine the four scalar fields into two complex fields

$$A = x_6 + ix_7, \quad B = x_8 + ix_9. \quad (3.29)$$

In the CHS geometry (3.2), A and B take the form

$$A = \sqrt{k} e^{Q\phi} \cos \theta e^{i\phi_2}, \quad B = \sqrt{k} e^{Q\phi} \sin \theta e^{i\phi_1}. \quad (3.30)$$

With the use of (3.30) we can recast (3.28) into the more suggestive form

$$\mathcal{L}_{\text{D5}} = \mathcal{L}_{\text{D5}} \Big|_{\lambda_{\pm}=0} - \frac{i}{k} \frac{\lambda_- A^* \partial_x A + \lambda_+ B^* \partial_x B}{\sqrt{1 + e^{-2Q\phi} (|\partial_x A|^2 + |\partial_x B|^2)}} + \mathcal{O}(\lambda_{\pm}^2) . \quad (3.31)$$

Furthermore, at sufficiently low energies $E \ll \frac{1}{R_X}$ we can expand the square root in the denominator in powers of $\partial_x A$ and $\partial_x B$. The leading order expansion gives

$$\mathcal{L}_{\text{D5}} \simeq \mathcal{L}_{\text{D5}} \Big|_{\lambda_{\pm}=0} - \frac{i}{k} (\lambda_- A^* \partial_x A + \lambda_+ B^* \partial_x B) . \quad (3.32)$$

As we will show in subsection 3.2, this result can be understood directly from the holographic prescription. First we show that the expression (3.32) is in perfect agreement with the perturbative expansion of the scaling dimensions (2.31) in the bulk.

Using the Fourier expansion of the Higgs fields A and B

$$A = \sum_{n \in \mathbb{Z}} A_n e^{-inx/R_X} , \quad B = \sum_{n \in \mathbb{Z}} B_n e^{-inx/R_X} \quad (3.33)$$

we can rewrite (3.32) in six dimensions as

$$\mathcal{L}_{\text{D5}} \simeq \mathcal{L}_{\text{D5}} \Big|_{\lambda_{\pm}=0} - \frac{1}{k} \sum_{n \in \mathbb{Z}} \frac{n}{R_X} (\lambda_- |A_n|^2 + \lambda_+ |B_n|^2) . \quad (3.34)$$

Taking into account the normalization of the kinetic terms, this result yields the mass shifts

$$\delta M^2(A_n) = -\frac{2n}{R_X k} \lambda_- , \quad \delta M^2(B_n) = -\frac{2n}{R_X k} \lambda_+ . \quad (3.35)$$

At the same time, we know from the bulk calculation (2.31) that a general mode with $p_w = 0$ and $p_n = \frac{n}{R_X}$ will exhibit the leading order mass shift⁸

$$\delta M_n^2 = -\frac{2n}{R_X} (\tilde{\lambda}_+ q_n + \tilde{\lambda}_- q_w) . \quad (3.36)$$

As explained in [30] the $U(1)_A \times U(1)_B$ rotation symmetries of the A, B planes are embedded in the $SU(2)_L \times SU(2)_R$ symmetry of the CHS background in the following way. The generator of $U(1)_A$ can be taken as $K_3^{(tot)} - \bar{K}_3^{(tot)}$, and the generator of $U(1)_B$ as $K_3^{(tot)} + \bar{K}_3^{(tot)}$. The corresponding charges can be normalized so that

$$\begin{aligned} (K_3^{(tot)} + \bar{K}_3^{(tot)})(A) &= 0 , & (K_3^{(tot)} - \bar{K}_3^{(tot)})(A) &= 1 , \\ (K_3^{(tot)} + \bar{K}_3^{(tot)})(B) &= 1 , & (K_3^{(tot)} - \bar{K}_3^{(tot)})(B) &= 0 . \end{aligned} \quad (3.37)$$

⁸ We set $p_w = 0$ here, because the DBI action only captures field theory effects.

As a result, single-trace gauge theory operators of the form $\text{tr}(A_n^k)$, $\text{tr}(B_n^k)$ correspond respectively to the string theory winding and momentum $\mathcal{N} = 2$ Liouville-type Kaluza Klein vertex operators

$$\mathcal{V}_A = e^{-\varphi - \bar{\varphi}} e^{i \frac{n}{R_X} (X + \bar{X})} e^{-\frac{1}{Q} (\phi - i(Y + \bar{Y}))} , \quad \mathcal{V}_B = e^{-\varphi - \bar{\varphi}} e^{i \frac{n}{R_X} (X + \bar{X})} e^{-\frac{1}{Q} (\phi - i(Y - \bar{Y}))} . \quad (3.38)$$

Applying the formula (3.36) to these modes and taking into account the redefinition (3.10) we find the mass shifts

$$\delta M^2(\mathcal{V}_A) = -\frac{2n}{R_X k} \lambda_- k , \quad \delta M^2(\mathcal{V}_B) = -\frac{2n}{R_X k} \lambda_+ k , \quad (3.39)$$

which are fully consistent with the DBI result (3.35).

In addition, the bulk computation of scaling dimensions predicts a second order mass shift, which is not expected a priori to appear in the same form in the dual gauge theory. For the lowest Kaluza-Klein modes ($p_n = p_w = 0$) the mass shift predicted by (2.31) (see also (2.30)) is

$$\delta M^2 = \left(\tilde{\lambda}_+ q_n + \tilde{\lambda}_- q_w \right)^2 . \quad (3.40)$$

Applying this formula to the vertex operators (3.38) (with Kaluza Klein momentum $n = 0$), we find a result that, up to a factor of k , implies in gauge theory the positive mass squared perturbation

$$\frac{\lambda_-^2}{2k^2} |A|^2 + \frac{\lambda_+^2}{2k^2} |B|^2 . \quad (3.41)$$

This result can also be reproduced from the DBI analysis. Indeed, the potential that follows from the action (3.21) is

$$\begin{aligned} U_{\text{D5}} &= e^{2Q\phi} \left[\left(1 - \frac{\lambda_+^2}{k} \right)^{-1} \sqrt{1 + \frac{\lambda_-^2 - \lambda_+^2}{k} \cos^2 \theta} - \left(1 - \frac{\lambda_+^2}{k} \right)^{-\frac{1}{2}} \right] \\ &= e^{2Q\phi} \left[\frac{\lambda_+^2}{2k} \sin^2 \theta + \frac{\lambda_-^2}{2k} \cos^2 \theta \right] + \mathcal{O}(\lambda_\pm^4) = \frac{\lambda_-^2}{2k^2} |A|^2 + \frac{\lambda_+^2}{2k^2} |B|^2 + \mathcal{O}(\lambda_\pm^4) . \end{aligned} \quad (3.42)$$

The zeroth order potential is vanishing as expected by supersymmetry at the supersymmetric point, but the next order contribution is quadratic and agrees with the bulk expectation (3.41). As we discuss in the next subsection, this is a rather non-trivial check of the holographic duality in this non-supersymmetric context.

3.2. Discussion

According to the general bulk-boundary correspondence the bulk gauge fields $(\mathcal{A}_{L,R})_\mu^a$ (see above (2.11)) correspond in the dual non-gravitational theory to a set of global Noether currents. As a result, the asymmetric bulk deformation (2.11) maps on the gauge theory (fivebrane) side to the Lagrangian deformation⁹

$$\delta\mathcal{L}_{D5} \propto \lambda_-(J_R - J_L) + \lambda_+(J_R + J_L) , \quad (3.43)$$

where $J_{L,R}$ are the $U(1)_{L,R}$ R-symmetry currents¹⁰

$$\begin{aligned} J_R &= \frac{i}{2}(A^* \partial_x A - \partial_x A^* A) + \frac{i}{2}(B^* \partial_x B - \partial_x B^* B) , \\ J_L &= -\frac{i}{2}(A^* \partial_x A - \partial_x A^* A) + \frac{i}{2}(B^* \partial_x B - \partial_x B^* B) . \end{aligned} \quad (3.44)$$

As stated earlier, (3.43), (3.44) reproduce the last part in (3.32).

An alternative way to understand this result relies on the use of the correspondence between states in the CHS geometry (2.1) and chiral primary operators in the dual low energy SYM theory, studied in [17,19]. According to the general dictionary, the worldsheet vertex operator $\partial Y \bar{\partial} Y$ is dual to the symmetric traceless operator

$$\text{tr}(A^* A - B^* B) . \quad (3.45)$$

When we add this operator to the SYM Lagrangian, one of the complex Higgs fields becomes massive and the other tachyonic.

One can show that the asymmetric deformations $\int d^2 z \mathcal{O}_\pm$ belong to the same supersymmetry multiplet as $\int d^2 z \partial Y \bar{\partial} Y$. Indeed, observe that

$$\begin{aligned} Q_+^2 Q_+^1 \cdot \partial Y &= \sqrt{\frac{1}{k}} Q_+^2 Q_+^1 \cdot \partial H_4 = -\frac{i}{2\sqrt{k}} e^{-\varphi+iH_2} , \\ Q_+^1 Q_+^2 \cdot \partial Y &= \sqrt{\frac{1}{k}} Q_+^1 Q_+^2 \cdot \partial H_4 = -\frac{i}{2\sqrt{k}} e^{-\varphi-iH_2} . \end{aligned} \quad (3.46)$$

$Q_\alpha^i, Q_{\dot{\alpha}}^i$ ($i = 1, 2$, $\alpha, \dot{\alpha} = \pm$) are spacetime supercharges and H_2, H_4 are bosons that bosonize the appropriate worldsheet fermions (see appendix A for a complete list of our

⁹ We thank David Kutasov for pointing this out to us.

¹⁰ In these expressions we set the fermions to zero.

conventions and the explicit form of the spacetime supercharges). Therefore, the vertex operator

$$e^{-\varphi}\psi^X = \frac{1}{\sqrt{2}}e^{-\varphi}(e^{iH_2} + e^{-iH_2}) , \quad (3.47)$$

which is the (-1) -picture version of ∂X , can be rewritten as

$$e^{-\varphi}\psi^X = i\sqrt{2k}(Q_+^2 Q_+^1 + Q_+^1 Q_+^2) \cdot \partial Y . \quad (3.48)$$

In a similar fashion for the right-movers, one can show that

$$e^{-\bar{\varphi}}\bar{\psi}^X = i\sqrt{2k}(\bar{Q}_+^2 \bar{Q}_+^1 + \bar{Q}_+^1 \bar{Q}_+^2) \cdot \bar{\partial} Y . \quad (3.49)$$

Combining these results and using the equivalence between the (-1) -picture and 0-picture vertex operators, we deduce for \mathcal{O}_\pm (see eq. (2.10))

$$\mathcal{O}_\pm = i\sqrt{2k}\left(Q_+^2 Q_+^1 + Q_+^1 Q_+^2 \pm (\bar{Q}_+^2 \bar{Q}_+^1 + \bar{Q}_+^1 \bar{Q}_+^2)\right) \cdot \partial Y \bar{\partial} Y . \quad (3.50)$$

Finally, by using the correspondence between the vertex operator $\partial Y \bar{\partial} Y$ and the gauge theory operator (3.45) we find that the gauge theory dual of \mathcal{O}_\pm is the descendant

$$i\left(Q_+^2 Q_+^1 + Q_+^1 Q_+^2 \pm (\bar{Q}_+^2 \bar{Q}_+^1 + \bar{Q}_+^1 \bar{Q}_+^2)\right) \text{tr}(A^* A - B^* B) . \quad (3.51)$$

Besides terms involving fermions, (3.51) includes the gauge theory operators

$$\text{tr}(A^* \partial_x A) \text{ and } \text{tr}(B^* \partial_x B) , \quad (3.52)$$

thus verifying the results arising from the DBI analysis (3.32) and the scaling dimension analysis (2.31) at leading order in the deformation parameters λ_\pm .

At second order in the deformation, both the bulk analysis (2.31) and the DBI analysis (3.41) suggest that we should add to the gauge theory Lagrangian the single trace operator

$$\lambda_-^2 \text{tr}(A^* A) + \lambda_+^2 \text{tr}(B^* B) . \quad (3.53)$$

This operator is symmetric, but not traceless. As a non-chiral primary operator in a non-supersymmetric theory, it is not a priori expected to appear simultaneously on both sides of the duality, especially beyond the leading order of the deformation. Here we find that it does.

4. Supergravity description of rotating fivebranes

So far we have discussed the effect of the asymmetric deformations (2.9) on the asymptotic CHS background (2.1). Because of the linear dilaton, this background exhibits a strong coupling singularity at $\phi \rightarrow -\infty$. This singularity can be resolved by adding to the worldsheet Lagrangian the $\mathcal{N} = 2$ Liouville interaction, which in superfield notation is

$$\delta\mathcal{S} = \mu \int d^2z d^2\theta \, e^{-\frac{1}{Q}(\phi+iY)} + c.c. \quad (4.1)$$

An alternative way to resolve the singularity is to replace the $\mathbb{R}_\phi \times S^1$ part of the geometry (2.3) with the $\mathcal{N} = 2$ Kazama-Suzuki supercoset $SL(2)_k/U(1)$ [31] at level $k = \alpha'/Q^2$. The target space of this conformal field theory has a cigar-shaped geometry and provides a geometric cut-off to the strong coupling singularity.

The $\mathcal{N} = 2$ Liouville theory (4.1) and the $\mathcal{N} = 2$ Kazama-Suzuki model are known to be equivalent by mirror symmetry [32]. In terms of NS5-branes both deformations take us into the Higgs branch of the theory, where the NS5-branes are separated symmetrically along a circle in the transverse space in an appropriate double scaling limit [18].

In the presence of the asymmetric deformations (2.9) the $\mathcal{N} = 2$ Liouville potential (4.1) is an irrelevant interaction on the worldsheet. The moduli space of the fivebranes has been lifted and k parallel NS5-branes at arbitrary positions do not any longer constitute a static configuration. On the worldsheet, a generic configuration will be captured by a time-dependent interaction. A natural guess is

$$\delta\mathcal{S} = \mu \int d^2z d^2\theta \, e^{i\omega t - \frac{1}{Q}(\phi+iY)} + c.c. \quad (4.2)$$

This is classically marginal when the frequency ω is

$$\omega^2 = \frac{1}{Q^2} \frac{\lambda_+^2}{1 - \lambda_+^2} . \quad (4.3)$$

Interestingly, this expression is λ_- -independent. Notice that by defining a new boson y via the relation

$$\frac{1}{Q}y = -\omega t + \frac{1}{Q}Y \quad (4.4)$$

we can recast (4.2) as the usual $\mathcal{N} = 2$ Liouville interaction

$$\delta\mathcal{S} = \mu \int d^2z d^2\theta \, e^{-\frac{1}{Q}(\phi+iy)} + c.c. \quad (4.5)$$

The new boson y is canonically normalized provided that

$$\partial Y(z)\partial Y(0) \sim -\frac{1}{1-\lambda_+^2} \log z \quad (4.6)$$

in the presence of the asymmetric deformation (2.9). We have verified this OPE explicitly for the case of generic λ_+ and $\lambda_- = 0$. Thus, we have a strong indication that the time-dependent deformation (4.2) is actually exactly marginal. In spacetime it describes a circular array of fivebranes rotating with constant angular velocity in the presence of the deformation (2.9).

It is interesting to ask if there is a corresponding description of rotating fivebranes in the language of a deformed $\mathcal{N} = 2$ Kazama-Suzuki supercoset $SL(2)_k/U(1)$. In the rest of this section, we will address this problem in the limit of large k , where the spacetime curvature is everywhere small and we can use a supergravity description.

Before the asymmetric deformation (2.9) the Kazama-Suzuki resolution of the background (2.3) is given by the exact conformal field theory

$$\mathbb{R}^{5,1} \times \left(\frac{SL(2)_k}{U(1)} \times \frac{SU(2)_k}{U(1)} \right) / \mathbb{Z}_k . \quad (4.7)$$

The string frame metric, B -field and dilaton for this solution are

$$ds^2 = dx_{||}^2 + dx^2 + k(d\rho^2 + \tanh^2 \rho d\tilde{\phi}_1^2 + d\theta^2 + \tan^2 \theta d\tilde{\phi}_2^2) , \quad (4.8)$$

$$B = 0 , \quad g_s^2 = \frac{1}{k \cos^2 \theta \cosh^2 \rho} . \quad (4.9)$$

At $\rho \rightarrow \infty$ this background asymptotes to (3.5). Before proceeding any further, we should raise the following word of caution. The supergravity solution (4.8), (4.9) belongs to a well-known list of examples [33-35] where the underlying theory is expected to be supersymmetric, because of extended worldsheet supersymmetry, but the supergravity solution is manifestly not supersymmetric. For that reason, it was argued in [36] that the supergravity solution (4.8), (4.9) is not the correct low-energy effective description of (4.7). The correct description is the T-dual background, which asymptotes to the CHS solution (2.1) and is manifestly supersymmetric. Having said this, our strategy in the ensuing will be the following. We start by analyzing the effects of the asymmetric deformations (2.9) on (4.8), (4.9) and then after a series of manipulations we T-dualize to a background that asymptotes to the deformed CHS solution. As we will see, this approach gives results which are in qualitative agreement with the picture implied by the analysis of the previous

sections. A similar approach for the case of the $\int d^2z \partial Y \bar{\partial} Y$ deformation was adopted in [12].

As we showed in the previous section, the asymptotic form of the $\lambda_+ \mathcal{O}_+ + \lambda_- \mathcal{O}_-$ deformation of (4.8), (4.9) is¹¹

$$ds^2 = -(dx^0)^2 + dx^2 + k(d\rho^2 + d\theta^2 + d\tilde{\phi}_1^2 + \tan^2 \theta d\tilde{\phi}_2^2) + 2\lambda_+ dx d\tilde{\phi}_1, \quad (4.10)$$

$$B = \lambda_- dx \wedge d\tilde{\phi}_1, \quad g_s^2 = \frac{2}{k \cos^2 \theta} e^{-2\rho} \left(1 - \frac{\lambda_+^2}{k}\right)^{\frac{1}{2}}. \quad (4.11)$$

For pedagogical reasons, in this section we will focus on the \mathcal{O}_- deformation, *i.e.* we will set $\lambda_+ = 0$ and $\lambda_- = \lambda$. This simple case is rather instructive and captures the basic features of the generic situation. More comments on the general λ_{\pm} deformation will appear at the end of this section.

It is convenient to T-dualize the asymptotic solution (4.10), (4.11) along the direction $\tilde{\phi}_1$. For $\lambda_+ = 0$ we get

$$ds^2 = -(dx^0)^2 + \left(1 + \frac{\lambda^2}{k}\right) dx^2 + 2\frac{\lambda}{k} dx d\tilde{\phi}_1 + k(d\rho^2 + d\theta^2 + \tan^2 \theta d\tilde{\phi}_2^2) + \frac{1}{k} d\tilde{\phi}_1^2, \quad (4.12)$$

$$B = 0, \quad g_s^2 = \frac{2}{k \cos^2 \theta} e^{-2\rho}. \quad (4.13)$$

The T-duality has allowed us to convert the non-zero B -field to an off-diagonal component of the metric. Then we can diagonalize the metric by using the coordinate transformation

$$\sqrt{1 + \frac{\lambda^2}{k}} x = \frac{1}{\sqrt{2}} (X + \Phi), \quad \frac{1}{\sqrt{k}} \tilde{\phi}_1 = \frac{1}{\sqrt{2}} (X - \Phi) \quad (4.14)$$

to obtain

$$\left(1 + \frac{\lambda^2}{k}\right) dx^2 + 2\frac{\lambda}{k} dx d\tilde{\phi}_1 + \frac{1}{k} d\tilde{\phi}_1^2 = \alpha_+ dX^2 + \alpha_- d\Phi^2 \quad (4.15)$$

where we have defined

$$\alpha_{\pm} \equiv 1 \pm \frac{\lambda}{k} \sqrt{\frac{k}{1 + \frac{\lambda^2}{k}}}. \quad (4.16)$$

Finally, it will be useful to define the new time coordinate t as

$$t = \frac{1}{\sqrt{\alpha_-}} x^0, \quad (4.17)$$

¹¹ In this section we explicitly include the $(dx^0)^2$ part of the metric in our formulae, because time will play a crucial rôle in what follows. The remaining four worldvolume coordinates do not participate in our manipulations and will be left implicit.

and perform the boost¹²

$$\Phi_{\text{new}} = C\Phi + St, \quad t_{\text{new}} = Ct + S\Phi, \quad C^2 - S^2 = 1. \quad (4.18)$$

For now the boost parameter C is an arbitrary number. It will be fixed uniquely in what follows by requiring that we get a regular background without conical singularities. In the new coordinates the background (4.12), (4.13) takes the form

$$ds^2 = -\alpha_- dt_{\text{new}}^2 + \alpha_+ dX^2 + \alpha_- d\Phi_{\text{new}}^2 + k(d\rho^2 + d\theta^2 + \tan^2 \theta d\tilde{\phi}_2^2), \quad (4.19)$$

$$B = 0, \quad g_s^2 = \frac{2}{k \cos^2 \theta} e^{-2\rho}. \quad (4.20)$$

We deform this solution in the following way

$$\begin{aligned} ds^2 = & -\alpha_- dt_{\text{new}}^2 + \frac{1}{2}(\coth^2 \rho + 1)(\alpha_+ dX^2 + \alpha_- d\Phi_{\text{new}}^2) - \\ & - \frac{\sqrt{\alpha_+ \alpha_-}}{\sinh^2 \rho} dX d\Phi_{\text{new}} + k(d\rho^2 + d\theta^2 + \tan^2 \theta d\tilde{\phi}_2^2), \end{aligned} \quad (4.21)$$

$$B = 0, \quad g_s^2 = \frac{1}{k \cos^2 \theta \sinh^2 \rho}. \quad (4.22)$$

This background is special, because

- (a) It is an exact solution of the lowest order α' equations of motion. To verify this property perform the change of coordinates

$$\sqrt{\alpha_+} X = \frac{1}{\sqrt{2}}(y + z), \quad \sqrt{\alpha_-} \Phi_{\text{new}} = \frac{1}{\sqrt{2}}(y - z) \quad (4.23)$$

to obtain

$$\begin{aligned} kd\rho^2 + \frac{1}{2}(\coth^2 \rho + 1)(\alpha_+ dX^2 + \alpha_- d\Phi_{\text{new}}^2) - \frac{\sqrt{\alpha_+ \alpha_-}}{\sinh^2 \rho} dX d\Phi_{\text{new}} = \\ = dy^2 + \coth^2 \rho dz^2 + kd\rho^2. \end{aligned} \quad (4.24)$$

This is the well-known trumpet solution, which describes the $U(1)$ vector gauging of $SL(2, \mathbb{R})$.

- (b) At $\rho \rightarrow \infty$ eqs. (4.21) and (4.22) reduce respectively to (4.19), (4.20) as required.

¹² In (4.15) X and Φ appear on equal footing. Hence, we could equally well perform the boost on the (x^0, X) plane. The meaning of this choice will be clarified below.

(c) For $\lambda = 0$ (and $C = 1, S = 0$), we will see in a moment that after the appropriate manipulations (4.21), (4.22) give rise to the anticipated cigar deformation (4.8) of the asymptotic geometry (3.5).

The next step is to re-express (4.21), (4.22) in terms of the original coordinates $(x, \tilde{\phi}_1)$ using (4.18), (4.14). With straightforward algebra we find the metric

$$\begin{aligned}
ds^2 = & \left(1 + \frac{\lambda^2}{k}\right) \left[1 + \frac{(\sqrt{\alpha_+} - \sqrt{\alpha_-}C)^2}{4\sinh^2\rho}\right] dx^2 + \frac{1}{k} \left[1 + \frac{(\sqrt{\alpha_+} + \sqrt{\alpha_-}C)^2}{4\sinh^2\rho}\right] d\tilde{\phi}_1^2 \\
& + \frac{1}{\sqrt{k}} \sqrt{1 + \frac{\lambda^2}{k}} \left[\alpha_+ - \alpha_- + \frac{\alpha_+ - \alpha_- C^2}{2\sinh^2\rho}\right] dx d\tilde{\phi}_1 \\
& + \frac{S}{\sqrt{2}\sinh^2\rho} \sqrt{1 + \frac{\lambda^2}{k}} (C\alpha_- - \sqrt{\alpha_+\alpha_-}) dx dt - \frac{S}{\sqrt{2k}\sinh^2\rho} (C\alpha_- + \sqrt{\alpha_+\alpha_-}) dt d\tilde{\phi}_1 \\
& - \alpha_- \left[1 - \frac{S^2}{2\sinh^2\rho}\right] dt^2 + k(d\rho^2 + d\theta^2 + \tan^2\theta d\tilde{\phi}_2^2) .
\end{aligned} \tag{4.25}$$

The B -field and dilaton are still given by (4.22). This solution has a strong curvature and g_s coupling singularity at $\rho = 0$, but soon we will T-dualize to a regular background.

As a trivial check, notice that when $\lambda = 0$ (and $C = 1, S = 0$) we have $\alpha_+ = \alpha_- = 1$ and the metric (4.25) becomes

$$ds^2 = -dt^2 + dx^2 + \frac{1}{k} \coth^2\rho d\tilde{\phi}_1^2 + k(d\rho^2 + d\theta^2 + \tan^2\theta d\tilde{\phi}_2^2) , \tag{4.26}$$

which can be T-dualized back along $\tilde{\phi}_1$ to obtain the cigar geometry

$$ds^2 = -dt^2 + dx^2 + k(d\rho^2 + \tanh^2\rho d\tilde{\phi}_1^2 + d\theta^2 + \tan^2\theta d\tilde{\phi}_2^2) . \tag{4.27}$$

This is precisely the deformation we wanted to obtain (see eq. (4.8)).

Next, we T-dualize (4.25) along the direction $\tilde{\phi}_1$. This gives rise to a background of the form

$$ds^2 = \sum_{r,s} \left[G_{rs} - \frac{G_{r\tilde{\phi}_1} G_{s\tilde{\phi}_1}^2}{G_{\tilde{\phi}_1\tilde{\phi}_1}} \right] dx^r dx^s + \frac{1}{G_{\tilde{\phi}_1\tilde{\phi}_1}} d\tilde{\phi}_1^2 + k(d\rho^2 + d\theta^2 + \tan^2\theta d\tilde{\phi}_2^2) , \tag{4.28}$$

$$B = \sum_r \frac{G_{r\tilde{\phi}_1}}{G_{\tilde{\phi}_1\tilde{\phi}_1}} dx^r \wedge d\tilde{\phi}_1 , \tag{4.29}$$

and the corresponding dilaton is $g_s^2 = \frac{1}{k \cos^2\theta \sinh^2\rho} \frac{\det G_{\text{new}}}{\det G_{\text{old}}}$. In these expressions (and below) the indices r, s correspond to x, t and G_{ij} are the metric components of (4.25). Note that in

comparison to the original cigar background (4.8), (4.9), additional electric and magnetic field components have been turned on.

We now argue that this background is regular. A possible singularity can occur at $\rho = 0$. Expanding all the components around $\rho = 0$ we find that a possible $\frac{1}{\rho^2}$ divergence of the coefficients of $dx^2, dt^2, dxdt$ vanishes automatically (for any value of C) and hence that the background is indeed regular. The string coupling g_s is also finite and bounded from above everywhere. For the metric coefficient of $d\tilde{\phi}_1^2$ we find

$$\frac{1}{G_{\tilde{\phi}_1\tilde{\phi}_1}} = \frac{4k}{(\sqrt{\alpha_+} + C\sqrt{\alpha_-})^2} \rho^2 + \mathcal{O}(\rho^4) . \quad (4.30)$$

In order to avoid a conical singularity we have to set

$$\frac{4k}{(\sqrt{\alpha_+} + C\sqrt{\alpha_-})^2} = k . \quad (4.31)$$

Using the definitions (4.16), this fixes the boost parameter C to

$$C = \frac{2 - \sqrt{1 + \frac{\lambda}{k} \sqrt{\frac{k}{1 + \frac{\lambda^2}{k}}}}}{\sqrt{1 - \frac{\lambda}{k} \sqrt{\frac{k}{1 + \frac{\lambda^2}{k}}}}} . \quad (4.32)$$

Notice that by definition the boost parameter C has to be greater than one. This is true for (4.32) only if $\lambda \geq 0$. If $\lambda \leq 0$, we simply repeat the above analysis with a boost on the (t, X) (instead of (t, Φ)) plane.

By changing back to the coordinates (ϕ_1, ϕ_2) and a further T-duality along ϕ_2 we can obtain the background of \mathcal{O}_- -deformed rotating NS5-branes in the language of the original CHS throat. After the change of coordinates $\tilde{\phi}_1 = \phi_1 + \frac{\phi_2}{k}$, $\tilde{\phi}_2 = \frac{\phi_2}{k}$ we find

$$\begin{aligned} ds^2 = \sum_{r,s} \left[G_{rs} - \frac{G_{r\tilde{\phi}_1} G_{s\tilde{\phi}_1}^2}{G_{\tilde{\phi}_1\tilde{\phi}_1}} \right] dx^r dx^s + \frac{1}{G_{\tilde{\phi}_1\tilde{\phi}_1}} d\phi_1^2 + \frac{2}{k G_{\tilde{\phi}_1\tilde{\phi}_1}} d\phi_1 d\phi_2 + \\ + k(d\rho^2 + d\theta^2) + \left(\frac{1}{k^2 G_{\tilde{\phi}_1\tilde{\phi}_1}} + \frac{1}{k} \tan^2 \theta \right) d\phi_2^2 , \end{aligned} \quad (4.33)$$

$$B = \sum_r \frac{G_{r\tilde{\phi}_1}}{G_{\tilde{\phi}_1\tilde{\phi}_1}} dx^r \wedge \left(d\phi_1 + \frac{1}{k} d\phi_2 \right) \quad (4.34)$$

with g_s^2 unchanged.

A further T-duality transformation along the direction ϕ_2 gives rise to our final rotating NS5-brane background, with metric

$$ds^2 = \sum_{\mu=1,2,3,4} (dx^\mu)^2 + \sum_{\alpha,\beta=x,t,\phi_1,\phi_2} g_{\alpha\beta} dx^\alpha dx^\beta + k(d\rho^2 + d\theta^2) , \quad (4.35)$$

$$g_{rs} = G_{rs} - \frac{G_{r\tilde{\phi}_1} G_{s\tilde{\phi}_1}}{G_{\tilde{\phi}_1\tilde{\phi}_1}} \left(1 - \frac{1}{\mathcal{G}}\right) , \quad g_{r\phi_2} = \frac{k G_{r\tilde{\phi}_1}}{\mathcal{G}} , \quad (4.36)$$

$$g_{\phi_1\phi_1} = \frac{1}{G_{\tilde{\phi}_1\tilde{\phi}_1}} \left(1 - \frac{1}{\mathcal{G}}\right) , \quad g_{\phi_2\phi_2} = \frac{k^2 G_{\tilde{\phi}_1\tilde{\phi}_1}}{\mathcal{G}} , \quad (4.37)$$

$$g_{r\phi_1} = g_{\phi_1\phi_2} = 0 , \quad (4.38)$$

and B -field components

$$B_{r\phi_1} = \frac{G_{r\tilde{\phi}_1}}{G_{\tilde{\phi}_1\tilde{\phi}_1}} \left(1 - \frac{1}{\mathcal{G}}\right) , \quad B_{\phi_1\phi_2} = \frac{k}{\mathcal{G}} , \quad (4.39)$$

$$B_{rs} = B_{r\phi_2} = 0 . \quad (4.40)$$

In these expressions G_{ij} refer to the metric components of (4.25), with α_\pm given by (4.16) and C fixed by the regularity constraint (4.32). The indices r, s correspond to x, t and we have defined the auxiliary function

$$\mathcal{G} \equiv 1 + k \tan^2 \theta G_{\tilde{\phi}_1\tilde{\phi}_1} . \quad (4.41)$$

Finally, the string coupling of the resulting background is given by the formula

$$g_s^2 = \frac{k}{\cos^2 \theta \sinh^2 \rho} \frac{\det g}{\det G} , \quad (4.42)$$

where $\det g$ is the determinant of the 4×4 part of the new g -metric along the directions x, ϕ_1, t, ϕ_2 and $\det G$ is the determinant of the corresponding 4×4 part of the G -metric in (4.25) along the directions $x, \tilde{\phi}_1, t, \phi_2$.

We observe that the final background has vanishing metric component $g_{t\phi_1}$, but non-vanishing $g_{t\phi_2}$. This suggests (see (3.30)) that the above solution describes a bunch of fivebranes rotating in the A plane in agreement with the scaling dimension analysis of section 2 and the DBI effective action analysis of section 3 that suggest a massive deformation for the Higgs field A , but nothing of the sort for B . An analogous rotating solution in supergravity was obtained in [12] for the $\int d^2 z \partial Y \bar{\partial} Y$ deformation (see (A.27) in [12]).

Another interesting property of the rotating solution in the presence of the \mathcal{O}_- deformation are the non-vanishing g_{xt} and $g_{x\phi_2}$ components, which couple the worldvolume directions of the fivebranes to the transverse directions.

Analogous results can be obtained with the use of similar methods (a combination of coordinate transformations, boosts and T-dualities) for the more general $\lambda_+ \mathcal{O}_+ + \lambda_- \mathcal{O}_-$ deformation. In particular, for the special case of $\lambda_- = 0$ and λ_+ generic we find a regular solution that has vanishing component $g_{t\phi_2}$ and non-vanishing $g_{t\phi_1}$. Again in agreement with the analysis of sections 2 and 3, this describes a bunch of fivebranes rotating in the B plane. For generic non-vanishing deformation parameters λ_{\pm} both $g_{t\phi_1}$ and $g_{t\phi_2}$ are non-vanishing and the more involved solution describes rotation in both the A and B planes. For example, this is the case of the half-supersymmetry preserving deformations that have $|\lambda_+| = |\lambda_-|$.

5. Backreaction at one-loop and the cosmological constant

From the above analysis we learn that the worldsheet interactions (2.9) are exactly marginal deformations of type II string theory on the CHS solution (2.1) that break, in general, the spacetime supersymmetry completely. In the large k limit, supergravity is valid and the corresponding statement is the following. The type II supergravity action (3.1) has a manifold of solutions parametrized by the deformation parameters λ_{\pm} . On this manifold the generic solution is non-supersymmetric. The two special lines defined by the equation $|\lambda_+| = |\lambda_-|$ are the only exception to this statement. On these lines eight supersymmetries are restored.

In the previous sections, we gave an explicit construction of these solutions at all orders in the deformation parameters. We also found rotating geometries where the string coupling g_s is a finite tunable parameter that can be chosen as small as we like.

Everything in the above discussion has been at tree-level. Once we break supersymmetry, interesting new effects can arise at-one loop. Since there is no exact cancellation between bosons and fermions any longer, the theory will generate a non-vanishing one-loop contribution that appears in the low-energy effective action (3.1) as a cosmological constant term $Z(\lambda_-, \lambda_+)$. This term can have a non-trivial effect, especially in the case of coincident fivebranes, where a strong coupling singularity develops deep inside the throat. Even then, however, the effect will be less drastic and under control in the asymptotic weakly

coupled region. In the Higgs branch, where the strong coupling problem is ameliorated, the one-loop effect is less drastic and under control everywhere.

It is interesting to investigate this one-loop backreaction effect with a supergravity analysis in the large k limit. In the asymptotic region, which will be the region of interest here, we expect on general grounds no drastic effects. In particular, no instabilities or time-dependence can arise, because the asymptotic spectrum has a mass gap. Hence, on general grounds one should still expect a manifold of solutions parametrized by the deformation parameters λ_{\pm} . To determine more concretely the precise effect of the one-loop contribution, we can use an action of the form

$$\mathcal{S}_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[e^{-2\Phi} \left(R + 4(\partial\Phi)^2 - \frac{1}{12} F_3^2 \right) - \frac{1}{12} G_3^2 + Z \right]. \quad (5.1)$$

The induced cosmological constant Z will be a function of the deformation parameters λ_{\pm} , since it comes from the one-loop correction to the non-supersymmetric solutions.¹³ To capture the effect of the one-loop backreaction to leading order in $g_s \propto e^{-\phi/\sqrt{k}}$ we will treat Z in what follows as a field independent quantity parameterized by the deformation parameters λ_{\pm} . A more involved treatment is needed at higher orders in g_s .

Solving the equations of motion that follow from (5.1) we find the one-loop corrected solution to the deformed background (3.12)-(3.14),

$$\begin{aligned} ds^2 = & \left(1 + \frac{1}{2} \sqrt{k} \phi Z g_s^2 \right) dx_{||}^2 + \left(1 - 3 \sqrt{k} \phi Z g_s^2 \right) d\phi^2 \\ & + \left[1 + \left(\frac{k}{8} + \frac{3\sqrt{k} + 2\phi}{4\sqrt{k}} (k - \lambda_+^2) \right) Z g_s^2 + \left(1 + \frac{k}{8} Z g_s^2 \right) \frac{\lambda_-^2 - \lambda_+^2}{k} \cos^2 \theta \right] dx^2 \\ & + k \left(1 + \frac{k}{8} Z g_s^2 \right) (d\theta^2 + \sin^2 \theta d\phi_1^2 + \cos^2 \theta d\phi_2^2) \\ & + 2 \left(1 + \frac{k}{8} Z g_s^2 \right) (\lambda_+ \sin^2 \theta dx d\phi_1 + \lambda_- \cos^2 \theta dx d\phi_2), \end{aligned} \quad (5.2)$$

with the Kalb-Ramond two-form field and g_s^2 still given by (3.13), (3.14).

As expected, we notice that this is a time-independent solution with a warping of the longitudinal world-volume part of the metric. By construction, the one-loop correction gives a finite shift to the tree-level solution that drops exponentially at the asymptotic infinity.

¹³ We anticipate that $Z(\lambda_-, \lambda_+)$ is an even function of λ_- and λ_+ because the solution is invariant under $\lambda_- \rightarrow -\lambda_-$ provided we also transform $\phi_2 \rightarrow -\phi_2$ (or alternatively $\lambda_+ \rightarrow -\lambda_+$ along with $\phi_1 \rightarrow -\phi_1$). Moreover, we expect that Z is zero by supersymmetry when $|\lambda_-| = |\lambda_+|$.

6. More asymmetric deformations

Up to this point we have been discussing the properties of the asymmetric deformations (2.9) for the $d = 6$ case with $\mathcal{M} = SU(2)/U(1)$ in (1.2). A few possible extensions of this discussion are as follows.

One possibility is to consider a more general deformation of the form

$$\lambda_{+;\mu\nu} \int d^2z \mathcal{O}_+^{\mu\nu} + \lambda_{-;\mu\nu} \int d^2z \mathcal{O}_-^{\mu\nu} , \quad (6.1)$$

where

$$\mathcal{O}_\pm^{\mu\nu} = \partial x^\mu \bar{\partial} Y \pm \partial Y \bar{\partial} x^\nu , \quad \mu, \nu = 0, 1, \dots, 5 . \quad (6.2)$$

We do not expect new physics from these deformations, since they can be viewed as a combination of (2.9) and the $SO(5, 1)$ Lorentz symmetry group of the theory.

Various aspects of our discussion can be generalized easily to the other non-critical superstrings in (1.2) with dimensions $d = 2, 4$ and general compact manifolds \mathcal{M} . In particular, the whole discussion of scaling dimensions in section 2 is universal and does not depend on the details of the theory. Eqs. (2.28) - (2.31) will always be true and our conclusions for the stability of the corresponding non-supersymmetric deformations will remain unaltered. Certain details of the DBI analysis are case specific, but the main conclusions about the agreement between bulk and boundary should of course remain unchanged. It is also possible to obtain rotating supergravity solutions similar to those of section 4 as long as we can take an appropriate supergravity limit in the non-critical superstring.

A potentially more interesting case is the one analyzed recently in [14], where (1.1) reads

$$\mathbb{R}^{2,1} \times \mathbb{R}_\phi \times SU(2)_{k_1} \times SU(2)_{k_2} . \quad (6.3)$$

This theory, which appears in the near horizon geometry of k_1 and k_2 NS5-branes with a three dimensional intersection, exhibits a mass gap [14,37] and possesses a supergravity limit.

Finally, in this paper we have explored the moduli space of asymmetric deformations (see figure 1) around the supersymmetric type IIA and type IIB theories of references [9,26]. In a similar fashion, we may consider asymmetric deformations around any point in the moduli space presented in [12], including for example the type 0A and type 0B theories. The generic point in the overall moduli space is a non-supersymmetric theory with stability properties that depend crucially on the details of the theory.

7. Summary and interesting prospects

Spacetime supersymmetry can be broken continuously in non-critical superstring theories with appropriate current-current deformations on the worldsheet. The deformed theory exhibits a variety of interesting features: bulk or localized tachyons, a non-supersymmetric spectrum, time dependence, a tunable cosmological constant. In this paper we analyzed a set of asymmetric current-current deformations and showed that the lightest modes receive a non-negative mass squared shift, which precludes the generation of bulk or localized tachyons. In a six dimensional example, we verified this effect at leading order in the deformation by analyzing the deformation on the dual low-energy SYM theory. For lowest Kaluza-Klein modes the leading order effect appears at second order and involves a non-chiral quadratic operator of the Higgs fields.

As another consequence of the mass shifts, we found that the $\mathcal{N} = 2$ Liouville interaction becomes irrelevant on the worldsheet. The exactly marginal interaction is time dependent and describes a configuration of rotating NS5-branes. We analyzed this effect in the supergravity limit and found a manifold of rotating supergravity solutions at weak g_s coupling.

There is a number of interesting extensions of the results presented in this paper. First of all, it would be interesting to repeat the analysis of the $d = 6$, $\mathcal{M} = SU(2)/U(1)$ case for the type IIA case. The results of section 2 are similar for the type IIA case, but the details of holography are different. The type IIA low-energy holographic dual is the six-dimensional $(0, 2)$ SCFT and the strong coupling singularity of (2.1) is better studied in eleven dimensional M-theory. It would be interesting to verify certain statements about holography in this context. Another interesting direction is to study the effects of the asymmetric deformations in heterotic non-critical superstring theory (for recent work in this theory see [38]).

There is also a rich story of open string dynamics on spaces of the form

$$\mathbb{R}^{d-1,1} \times \left(\frac{SL(2)_k}{U(1)} \times \mathcal{M} \right) / \Gamma . \quad (7.1)$$

D-branes on such spaces have been analyzed in a series of recent papers (see [39] and references therein) and are expected to play a key rôle in uncovering the inner workings of LST's and the corresponding holographic dualities. The dynamics of these branes are also interesting from the gauge theory point of view. In recent work [40,41] it has been verified explicitly that there are appropriate D-brane configurations on (7.1) that realize

gauge theories with minimal supersymmetry. For instance, one can obtain four-dimensional $\mathcal{N} = 1$ SQCD with appropriate D-branes on

$$\mathbb{R}^{3,1} \times \frac{SL(2)_1}{U(1)} . \quad (7.2)$$

Further work on gauge theories in this general context can be found in [42].

It will be interesting to study the effect of the asymmetric deformations on D-branes on (7.1). This will give further information about the dynamics of the non-critical string on (7.1), and will, in particular, clarify how the breaking of supersymmetry in the bulk affects the dynamics of the gauge theory on the D-branes. For that purpose, it will be useful to obtain a better grasp on the exact CFT properties of the asymmetric deformations on (7.1) beyond the supergravity limit.

Since we break supersymmetry in a well controlled stringy environment it is tempting to ask if there are any potential phenomenological applications of our work. Perhaps this could be achieved along the lines of [43] and would be worth investigating further. A related question, which is also interesting on its own, has to do with the higher-loop backreaction problem. As a starting point in this direction, we analyzed in section 5 the leading order effects of the one-loop cosmological constant on the tree-level deformed CHS solutions. In the asymptotic region, where the coupling constant goes continuously to zero we determined the exact, finite, time-independent shift of the one-loop contribution to the tree-level solutions. It would be interesting to go further and give a more complete treatment of the backreaction problem in this setting.

Finally, it could be interesting to use similar techniques to further examine the thermodynamics of near-extremal NS5-brane backgrounds and their application to LST at finite temperature. It was recently shown in [44] that, in the canonical ensemble, the usual near-extremal NS5-brane background is subdominant to a new stable phase of near-extremal M5-branes localized on a transverse circle, having a limiting temperature that lies above the Hagedorn temperature. It is conceivable that there are other stable phases that are relevant to the thermodynamic behavior of LST.

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Appendix A. Summary of conventions

In this appendix we summarize the basic conventions used in the main text. For k parallel NS5-branes wrapped on a circle S^1 the near horizon geometry takes the form

$$\mathbb{R}^{4,1} \times S_X^1 \times \mathbb{R}_\phi \times SU(2)_k . \quad (\text{A.1})$$

The worldsheet theory on (A.1) comprises of six free scalars x^μ ($\mu = 0, 1, \dots, 5$), six corresponding free real fermions ψ^μ , one linear dilaton scalar ϕ with linear dilaton slope $Q = 1/\sqrt{k}$ (we set $\alpha' = 1$), the corresponding free fermion ψ^ϕ and finally the supersymmetric $SU(2)$ WZW model at level k . In the main text, we compactify the sixth direction and denote the corresponding boson and fermion as X and ψ^X respectively. The supersymmetric $SU(2)_k$ WZW model comprises of a bosonic $SU(2)_{k-2}$ WZW model at level $k-2$ and three real free fermions χ^\pm, χ^3 . We denote the three (left-moving) $SU(2)$ currents as K^\pm, K^3 .

It is convenient to bosonize the above fermions in the following manner

$$\begin{aligned} \frac{1}{\sqrt{2}}(\psi^1 \pm \psi^0) &= e^{\pm iH_0} , & \frac{1}{\sqrt{2}}(\psi^2 \pm i\psi^3) &= e^{\pm iH_1} \\ \frac{1}{\sqrt{2}}(\psi^X \pm i\psi^4) &= e^{\pm iH_2} , & \frac{1}{\sqrt{2}}(\psi^\phi \pm \chi^3) &= e^{\pm iH_3} , \end{aligned} \quad (\text{A.2})$$

and

$$\chi^\pm = e^{\pm iH_4} . \quad (\text{A.3})$$

We also define and bosonize the total $K_3^{(tot)}$ current as

$$K_3^{(tot)} = K^3 + \chi^+ \chi^- = K^3 + i\partial H_4 = i\sqrt{k}\partial Y . \quad (\text{A.4})$$

Notice that for $k = 2$ the bosonic $SU(2)_{k-2}$ WZW model becomes trivial and the boson H_4 equals $\sqrt{2}Y$.

The above theory has $\mathcal{N} = (4, 4)$ worldsheet supersymmetry. The construction of the spacetime supercharges makes use of the $U(1)_R$ current of the $\mathcal{N} = (2, 2)$ worldsheet supersymmetry, which in our case has the (left-moving) fermionic generators

$$G^\pm = \sqrt{2} \sum_{i=0}^2 e^{\mp i H_i} \partial x^i + Q \chi^\mp K^\pm + i \left(e^{\mp i H_3} (\partial \phi \pm i \partial Y) - Q \partial e^{\mp i H_3} \right) \quad (\text{A.5})$$

and the $U(1)_R$ current

$$J = -i \partial H_4 + i \partial H_3 . \quad (\text{A.6})$$

The three complex bosons x^i are defined as

$$x^0 = \frac{1}{\sqrt{2}}(x^0 + x^1) , \quad x^1 = \frac{1}{\sqrt{2}}(x^2 + i x^3) , \quad x = \frac{1}{\sqrt{2}}(X + i x^4) . \quad (\text{A.7})$$

Type IIB string theory on (A.1) exhibits sixteen supercharges in the

$$(\mathbf{4}, \mathbf{2}) + (\mathbf{4}', \mathbf{2}') \quad (\text{A.8})$$

representations of $SO(5, 1) \times SO(4)$. The eight supercharges Q_A^\pm in $(\mathbf{4}, \mathbf{2})$ arise from the left-moving sector of the string and the other eight supercharges $\bar{Q}_{\dot{A}}^\pm$ in $(\mathbf{4}', \mathbf{2}')$ arise from the right-moving sector. In worldsheet terms these supercharges can be written as

$$Q_A^\pm = \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2} \mp \frac{i}{2}(H_3 - H_4)} \mathcal{S}_A , \quad (\text{A.9})$$

$$\bar{Q}_{\dot{A}}^\pm = \oint \frac{d\bar{z}}{2\pi i} e^{-\frac{\bar{\varphi}}{2} \mp \frac{i}{2}(\bar{H}_3 - \bar{H}_4)} \bar{\mathcal{S}}_{\dot{A}} , \quad (\text{A.10})$$

where $\mathcal{S}_A, \bar{\mathcal{S}}_{\dot{A}}$ are the spin fields

$$\mathcal{S}_A = e^{\frac{i}{2}(\alpha_0 H_0 + \alpha_1 H_1 + \alpha_2 H_2)} , \quad \alpha_i = \pm 1 , \text{ even number of } -'s , \quad (\text{A.11})$$

and

$$\bar{\mathcal{S}}_{\dot{A}} = e^{\frac{i}{2}(\bar{\alpha}_0 \bar{H}_0 + \bar{\alpha}_1 \bar{H}_1 + \bar{\alpha}_2 \bar{H}_2)} , \quad \bar{\alpha}_i = \pm 1 , \text{ odd number of } -'s . \quad (\text{A.12})$$

From the five dimensional point of view the spacetime supercharges take the following form. Again, eight supercharges arise from the left-moving sector

$$Q_\alpha^1 = \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2} + \frac{i}{2}(H_2 + H_3 - H_4) + i \frac{\varphi}{2}(H_0 + H_1)} , \quad Q_{\dot{\alpha}}^1 = \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2} - \frac{i}{2}(H_2 + H_3 - H_4) + i \frac{\varphi}{2}(H_0 - H_1)} , \quad (\text{A.13})$$

$$Q_\alpha^2 = \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2} + \frac{i}{2}(-H_2+H_3-H_4) + i\frac{\alpha}{2}(-H_0+H_1)} , \quad Q_{\dot{\alpha}}^2 = \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2} + \frac{i}{2}(H_2-H_3+H_4) - i\frac{\dot{\alpha}}{2}(H_0+H_1)} , \quad (\text{A.14})$$

and eight more $\bar{Q}_\alpha^i, \bar{Q}_{\dot{\alpha}}^i$ ($i = 1, 2$) arise from the right-moving sector. In type IIB the latter are

$$\bar{Q}_\alpha^1 = \oint \frac{d\bar{z}}{2\pi i} e^{-\frac{\bar{\varphi}}{2} + \frac{i}{2}(\bar{H}_2+\bar{H}_3-\bar{H}_4) + i\frac{\alpha}{2}(\bar{H}_0-\bar{H}_1)} , \quad \bar{Q}_{\dot{\alpha}}^1 = \oint \frac{d\bar{z}}{2\pi i} e^{-\frac{\bar{\varphi}}{2} - \frac{i}{2}(\bar{H}_2+\bar{H}_3-\bar{H}_4) - i\frac{\dot{\alpha}}{2}(\bar{H}_0+\bar{H}_1)} , \quad (\text{A.15})$$

$$\bar{Q}_\alpha^2 = \oint \frac{d\bar{z}}{2\pi i} e^{-\frac{\bar{\varphi}}{2} + \frac{i}{2}(-\bar{H}_2+\bar{H}_3-\bar{H}_4) + i\frac{\alpha}{2}(\bar{H}_0+\bar{H}_1)} , \quad \bar{Q}_{\dot{\alpha}}^2 = \oint \frac{d\bar{z}}{2\pi i} e^{-\frac{\bar{\varphi}}{2} + \frac{i}{2}(\bar{H}_2-\bar{H}_3+\bar{H}_4) - i\frac{\dot{\alpha}}{2}(\bar{H}_0-\bar{H}_1)} . \quad (\text{A.16})$$

The indices $\alpha, \dot{\alpha}$ take the values \pm . The reduction from six to five dimensions works as follows

$$\begin{aligned} Q_A^+ &\rightarrow \{Q_{\dot{\alpha}}^1, Q_{\dot{\alpha}}^2\} , \quad Q_A^- \rightarrow \{Q_\alpha^1, Q_\alpha^2\} , \\ \bar{Q}_A^+ &\rightarrow \{\bar{Q}_{\dot{\alpha}}^1, \bar{Q}_{\dot{\alpha}}^2\} , \quad \bar{Q}_A^- \rightarrow \{\bar{Q}_\alpha^1, \bar{Q}_\alpha^2\} . \end{aligned} \quad (\text{A.17})$$

Appendix B. Summary of the Buscher rules

For quick reference, we briefly recall here the Buscher rules of T-duality (see for instance [28]). For a general background that has an isometry along the direction x^0 , metric of the form $(i, j, \dots \neq 0)$

$$ds^2 = G_{00}(dx^0)^2 + 2G_{0i}dx^0dx^i + 2G_{ij}dx^i dx^j \quad (\text{B.1})$$

and B -field components B_{0i}, B_{ij} the T-duality transformation along x^0 acts in the following manner

$$G'_{00} = \frac{1}{G_{00}} , \quad G'_{0i} = \frac{B_{i0}}{G_{00}} , \quad G'_{ij} = G_{ij} - \frac{G_{i0}G_{0j} - B_{0i}B_{0j}}{G_{00}} , \quad (\text{B.2})$$

$$B'_{i0} = \frac{G_{0i}}{G_{00}} , \quad B'_{ij} = B_{ij} - \frac{G_{0i}B_{0j} - B_{0i}G_{0j}}{G_{00}} . \quad (\text{B.3})$$

The dilaton transforms as

$$g_{s,\text{new}}^2 = g_{s,\text{old}}^2 \frac{\det G_{\text{new}}}{\det G_{\text{old}}} . \quad (\text{B.4})$$

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